Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Calculus 1501B First Midterm Examination

Friday, March , 2012 7:00 p.m. – 9:30 p.m.

INSTRUCTIONS

1. Do not unstaple the booklet. Do not tear any pages from the booklet.
2. Questions start on Page 1 and continue to Page 11. Questions are printed on both sides of the paper. BE SURE YOU HAVE A COMPLETE BOOKLET.
3. CALCULATORS AND NOTES ARE NOT PERMITTED.
4. SHOW ALL YOUR WORK. Answer all questions in the spaces provided.
5. TOTAL MARKS = 100.

Student Number (**Print**)

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FOR GRADING ONLY

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1. Consider the sequence defined by the recursion $a_{n+1} = \sqrt{a_n}$.

6 marks (a) Show that if $0 < a_0 < 1$, then a_n is convergent.

6 marks (b) Given that a_n is convergent and $0 < a_0 < 1$, evaluate $L = \lim_{n\to\infty} a_n$.

8 marks 2. Evaluate the sum of the (convergent) series $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n(n+1)}$.

6 marks 3. Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+1}$ is absolutely convergent, conditionally convergent or divergent. Justify your answer.

4. Let s denote the sum of the (convergent) infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$.

6 marks (a) It can be shown that $1 + \frac{1}{2^4} + \frac{1}{3^4} + \ldots + \frac{1}{10^4} = 1.0820\ldots$. Use this fact to derive an upper and lower bound for s.

6 marks (b) In (a) we used 10 terms to estimate s. How many terms would we need to use in order to ensure that the resulting error was no larger than $\frac{10^{-6}}{3}$?

8 marks 5. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+7}{\sqrt{n^3+3n-1}}$ converges or diverges.

8 marks 6. Determine whether the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n\, n!}{(2n)!}$ is absolutely convergent, conditionally convergent or divergent.

8 marks 7. Determine whether the series $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n\ln(n)}$ is absolutely convergent, conditionally convergent or divergent.

5 marks 8. (a) Is the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n+1}}$ conditionally or absolutely convergent? Justify your answer.

5 marks (b) How many terms would be required in order to estimate the sum of the series with an error that does not exceed 10^{-4}?

10 marks 9. Determine the radius and interval of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(2x-7)^n}{3n+1}$.

6 marks 10. (a) Express $\dfrac{x}{1+x^4}$ as a power series. Be sure to indicate the radius and interval of convergence.

6 marks (b) Estimate $\displaystyle\int_0^1 \frac{x}{1+x^4}\,dx$ using the third partial sum of an appropriate series.

6 marks 11. Suppose that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = -\frac{2}{3}$. Evaluate $\lim_{n\to\infty} a_n$.

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