1. Find the limit of the given sequence, or explain why it diverges. Justify your reasoning.

5 marks (a) $\left\{\frac{\sin 2n}{\cos n+\sqrt{n}}\right\}_{n=1}^{\infty}$

5 marks (b) $\{\sqrt{e}, \sqrt{e\sqrt{e}}, \sqrt{e\sqrt{e\sqrt{e}}}, \dots\}$

10 marks 2. Find the Taylor series for $f(x) = \cos x$ centred at $a = \pi$. Write your answer using the Σ-notation, but do not determine the radius or the interval of convergence of the series.

10 marks 3. Find the Cartesian coordinates of the point on the parametric curve

$$x = \cos^3 t, \quad y = 3\sin^3 t,$$

which is in the first quadrant and where the slope of tangent line to the curve equals $-\sqrt{3}$.

10 marks 4. A tank initally contains 200 litres (L) of pure water. Brine containing 2 kilograms (kg) of salt per 20 L is then added to the tank at a rate of $\frac{1}{2}$ L per hour (h). As brine added to the solution is kept well mixed and the brine solution is drained from the tank at rate $\frac{1}{2}$ L per h. How much salt (in kg) remains in the tank after 100 hours have passed?

10 marks 5. Evaluate

$$\int_0^{\infty} xe^{-2x}dx.$$

Explain carefully every step involved.

6.

5 marks (a) Find the point of self-intersection of the curve given by the parametric equations

$$x = 3t - t^3, \quad y = 3t^2.$$

5 marks (b) Find the length of the loop of the curve in Part (a).

10 marks 7. Solve the initial value problem

$$y' = \frac{e^y}{x^2 - 2x}, \quad y(1) = 0.$$

10 marks 8. Suppose that the function $f(x)$ is continuous on the interval $[1,2]$ and differentiable on $(1,2)$. Assume that $f(1) = 1$, and $f(2) = \frac{1}{2}$, and $f \neq 0$. Prove that there exists a point $c \in (1,2)$ such that

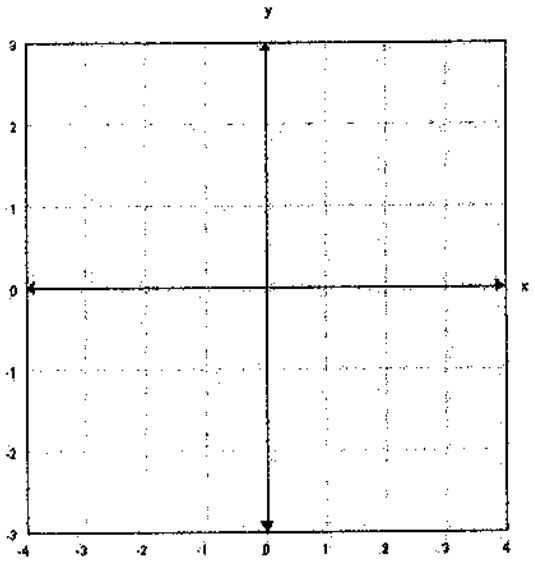
$$f'(c) = -[f(c)]^2.$$

(*Hint: Apply the Mean Value Theorem to the function* $1/f(x)$.)

10 marks 9. Find the first four non-zero terms of the Maclaurin series of the function $y = \frac{1}{\sqrt{1+x^2}}$.

10.

5 marks (a) Sketch the curve given by the parametric equation $r = 2\cos\theta$, where $0 \leq \theta \leq \pi$. Give the Cartesian coordinates of the intersection of this curve with the curve $r = 1$.



5 marks (b) Use a definite integral to calculate the area bounded by the curve $r = 2\cos\theta$.