10 marks 1. If $a_n = (-1)^n \dfrac{n}{n+1}$, is $\{a_n\}_{n=1}^{\infty}$ monotonic? Is it bounded? Explain.

5 marks 2. Let $a_n = \dfrac{1}{2^n}$. Use the formal ε, N definition to show that $\lim\limits_{n\to\infty} a_n = 0$.

5 marks 3. Use the Squeeze Theorem to determine $\lim_{n\to\infty} \dfrac{n+\ln n}{n^2+(-1)^n}$.

4.

5 marks (a) Give example of a series $\sum_{n=1}^{\infty} a_n$ such that $\lim_{n\to\infty} a_n = 0$, but the series diverges.

5 marks (b) Let $\sum_{n=1}^{\infty} a_n$ be a series with only positive terms, and let $S_N = \sum_{n=1}^{N} a_n$ be the partial sum of the first N terms of the series (i.e., the partial sum of order N). Prove that if $S_N < 5 - \sin(N^2)$, then the series $\sum a_n$ converges.

5. Determine if each of the following series is convergent or divergent. Be clear about any test for convergence/divergence you apply.

5 marks (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n^2}}{n!}$

5 marks (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$

6. Determine if each of the following series converges absolutely, converges conditionally, or diverges. Be clear about any test for convergence/divergence you apply.

5 marks (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{n}{1+\ln n}$

5 marks (b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(\ln n)^2}{n}$

10 marks 7. Use the power series representation of $\dfrac{1}{1-x}$ to evaluate $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{1}{3}\right)^{n-1}$.

10 marks 8. Given $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n\sqrt{n}}\,(x-1)^n$, what is the interval of convergence of the series?

9.

5 marks (a) Evaluate $\int e^{-x^2}\,dx$ as a power series centred at the origin. Write the first three nonzero terms of the series.

5 marks (b) Determine the interval of convergence of the power series found in Part (a) above.

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Instructor's Name (**Print**)

Student's Name (**Print**)

Student's Signature

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Calculus 1501 Second Midterm Examination

Friday, March 15, 2013 7:00 p.m. – 9:00 p.m.

INSTRUCTIONS

1. Do not unstaple the booklet. Do not tear any pages from the booklet.
2. Questions start on Page 1 and continue to Page 9. Questions are printed on both sides of the paper. BE SURE YOU HAVE A COMPLETE BOOKLET.
3. CALCULATORS AND NOTES ARE NOT PERMITTED.
4. SHOW ALL YOUR WORK. Answer all questions in the spaces provided.
5. TOTAL MARKS = 80.

Student Number (**Print**)

Student's Name (**Print**)

FOR GRADING ONLY

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1	
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