Group Theory 3120B, 1st Assignment
Due Thursday Feb 27th, 2014, in class

1. Let H and K be finite subgroups of a group G. (a) If the orders of H and K are relatively prime, show that $H \cap K = \{e\}$. (b) If the order of H is a prime show that either $H \cap K = \{e\}$ or $H \subset K$.

2. Let G be a group. (a) Show that for any a in G, $ord(a) = ord(a^{-1})$. (b) Show that for any a and b in G, $ord(a) = ord(bab^{-1})$. (c) If a, b have finite order in G, does it follow that ab is also of finite order? (d) Is there a general formula for $ord(ab)$ if a, b have finite order in G and $ab = ba$?

3. Show that if a group G has only one element a of order n, then $a \in Z(G)$ and $n = 2$.

4. Show that there does not exist a nonzero group homomorphism from S_3 to $\mathbb{Z}_3$.

5. Show that a group G has no proper subgroups if and only if it is a cyclic group of prime order.

6. Find a homomorphism from S_3 onto a nontrivial cyclic group.

7. Show that the cyclic subgroup of S_3 generated by $f : \{1, 2, 3\} \to \{1, 2, 3\}, f(1) = 2, f(2) = 1, f(3) = 3$, is not normal in S_3.

8. If N is a normal subgroup of a group G such that $N \cap G' = \{e\}$, show that $N \subset Z(G)$.

9. Show that there does not exist any group G such that $G/Z(G)$ is of order 37.

10. Find $Aut(\mathbb{Z}_n, +)$.