Group Theory 3120B, 2st Assignment
Due Friday March 14th, 2014

1. Characterize simple abelian groups.

2. (a) Show that $(\mathbb{Q}, +)$ and $(\mathbb{R}, +)$ do not have any maximal subgroups.
(b) Find all maximal subgroups of $(\mathbb{Q}^* = \mathbb{Q} \setminus \{0\}, \cdot)$.

3. (a) Let N be a normal subgroup of a group G such that $[G, N] = n < \infty$. Show that G/N is isomorphic to a subgroup of S_n.
(b) Show that if G is a simple group that has a proper subgroup of finite index n in G, then G is isomorphic to a subgroup of S_n.
(c) Let G be a finite group and H be a proper subgroup of G of index n in G. If $|G|$ is a not a divisor of $n!$, show that G is not simple.
(d) Show that any infinite group G which has a proper subgroup of finite index is not simple.

4. Find $Aut(S_3)$.

5. (a) Show that $S_n =< \{(1, 2, \ldots, n-1), (n-1, n)\} >$.
(b) Show that $S_n =< \{(1, 2, \ldots, n), (1, 2)\} >$.
(c) Is $S_n =< \{(1, 2), (1, 3), (1, 4) \ldots, (1, n)\} >$?

6. Show that the order of any $\sigma \in S_n$ is the least common multiple of the lengths of its disjoint cycles.

7. Show that two cycles in S_n are conjugate if and only if they are of the same length.

8. Prove that in any group, the set of all elements that have only a finite number of conjugates is a subgroup.

9. Let G be a group such that $|G'| = m$. Show that any element in G has at most m conjugates.

10. Show that if a finite group G has a normal subgroup N such that $|N| = 3$ and $N \not\subset Z(G)$, then G has a subgroup K of index 2 in G.