Lebesgue Integration and Fourier Series
Mathematics 4122B/9022B, Western University
Winter 2013

Time and place: Monday, Wednesday, Thursday, 10:30–11:30 a.m., MC 108.

Instructor: Farzad Fathizadeh, ffathiz@uwo.ca, MC 113.

Textbook: *Real Analysis: Measure Theory, Integration, and Hilbert Spaces,* by E. M. Stein and R. Shakarchi, Princeton University Press, 2005.

Course description: This course is intended to present basic facts about measure theory and integration. It will start by the construction of Lebesgue measure in Euclidean spaces. After introducing Lebesgue measurable subsets of $\mathbb{R}^d$ and their measure, Lebesgue integration will be discussed. Special attention will be paid to several convergence theorems and Fubini's theorem for Lebesgue integration, and the fact that it extends Riemann integration of continuous functions and leads to complete L^p spaces. These results will highlight that Lebesgue integration, compared to Riemann integration, provides a more flexible framework and naturally gives rise to Banach spaces that are studied vastly in functional analysis. We will then introduce the notion of an abstract measure space, give examples such as counting measure and Dirac measure, and will discuss the integration theory in the abstract context. We will see that in spite of its generality, integration theory on abstract measure spaces enjoys several convergence theorems and Fubini's theorem (already discussed for Lebesgue measure on $\mathbb{R}^d$). This explains why measure theory is applicable in a wide range of contexts and in fact plays a very important role in real analysis, probability theory, and the theory of partial differential equations. An important topic that will be covered in this course is the notion of absolute continuity and the Lebesgue-Radon-Nikodym theorem. Time permitting, some topics on the relation of differentiation to Lebesgue integration on $\mathbb{R}$, and also some topics from Hilbert space theory and the Fourier transform will be discussed.

Syllabus: The material described above will cover the main parts of Chapter 1: Measure Theory, Chapter 2: Integration Theory, Chapter 6: Abstract Measure and Integration Theory, and selected topics from Chapter 3: Differentiation and Integration, and Chapter 5: Hilbert Spaces: Several Examples.

Grading scheme: Two assignments 30% each, and final exam 40%.

Course website: Here is a link:
http://www.math.uwo.ca/ ffathiza/Courses/MeasureTheory/measure.html.