1.

a) $$\left(\frac{1}{16}\right)^{-1/4}\left(\frac{27}{64}\right)^{1/4} = \frac{1}{16^{-1/4}}\,\frac{27^{1/4}}{64^{1/4}} = 16^{1/4}\cdot\frac{27^{1/4}}{64^{1/4}}$$

$$= 2\cdot\frac{(3^3)^{1/4}}{(2^6)^{1/4}} = \frac{2\sqrt[4]{27}}{2^{3/2}} = \frac{2\sqrt[4]{27}}{2\sqrt{2}} = \frac{\sqrt[4]{27}}{\sqrt{2}}$$

b) $$\frac{5^{2.3}}{5^{-0.3}\,5^{1.2}} = \frac{5^{2.3}}{5^{-0.3+1.2}} = \frac{5^{2.3}}{5^{0.9}} = 5^{2.3-0.9} = 5^{1.4}$$

c) $$\log_3 81 = \log_3 3^4 = 4\log_3 3 = 4$$

d) $$\ln\frac{1}{e^{2/3}} = \ln e^{-2/3} = -\frac{2}{3}\ln e = -\frac{2}{3}.$$

2.

a) $3^{x-1} = \frac{1}{9^{2x}} \Rightarrow 3^{x-1} = \frac{1}{(3^2)^{2x}} \Rightarrow 3^{x-1} = \frac{1}{3^{4x}}$

$\Rightarrow 3^{4x} \cdot 3^{x-1} = 1 \Rightarrow 3^{5x-1} = 3^0 \Rightarrow 5x-1=0$

$\Rightarrow 5x = 1 \Rightarrow x = \frac{1}{5}$

b) $\log_3(t+4) + \log_3(t-4) = 1$

$\Rightarrow \log_3\left((t+4)\cdot(t-4)\right) = 1$

$\Rightarrow (t+4)(t-4) = 3^1 \Rightarrow t^2 - 16 = 3 \Rightarrow t^2 = 19$

$\Rightarrow t = \sqrt{19}$ or $t = -\sqrt{19}$.

$t = -\sqrt{19}$ cannot be a solution since $-\sqrt{19} + 4 < 0$ and its $\log_3$ does not make sense. So $t = \sqrt{19}$ is the only solution of the above equation.

3.

a) $\frac{d}{dx}\left(x^2 e^{x^2} + 5^{2x+1}\right)$

$= 2x\, e^{x^2} + x^2\left(2x\, e^{x^2}\right) + 2\left(\ln 5\right) 5^{2x+1}$

$= 2x\, e^{x^2} + 2x^3 e^{x^2} + 2 \cdot \ln 5 \cdot 5^{2x+1}$

$= 2\left(x + x^3\right) e^{x^2} + 2 \cdot \ln 5 \cdot 5^{2x+1}$

b) $\frac{d}{dx}\left(\ln(x^2+5x) + \frac{x}{e^x}\right)$

$= \frac{2x+5}{x^2+5x} + \frac{e^x - e^x x}{(e^x)^2} = \frac{2x+5}{x^2+5x} + \frac{e^x(1-x)}{e^{2x}}$

$= \frac{2x+5}{x^2+5x} + \frac{1-x}{e^x}$.

c) $\frac{d}{dx}\left(\log_5(e^x+5x)\right) = \frac{e^x+5}{(\ln 5)(e^x+5x)}$

4. Let $y = \dfrac{x^{1/2}(x-5)^7(x+6)^3}{(x+1)^2}$.

Then

$$\ln y = \ln \frac{x^{1/2}(x-5)^7(x+6)^3}{(x+1)^2}$$

$$\Rightarrow \ln y = \ln\left(x^{1/2}(x-5)^7(x+6)^3\right) - \ln\left((x+1)^2\right)$$

$$\Rightarrow \ln y = \ln x^{1/2} + \ln(x-5)^7 + \ln(x+6)^3 - \ln(x+1)^2$$

$$\Rightarrow \ln y = \frac{1}{2}\ln x + 7\ln(x-5) + 3\ln(x+6) - 2\ln(x+1).$$

So by taking the derivative $\frac{d}{dx}$ of both sides of the latter, we have

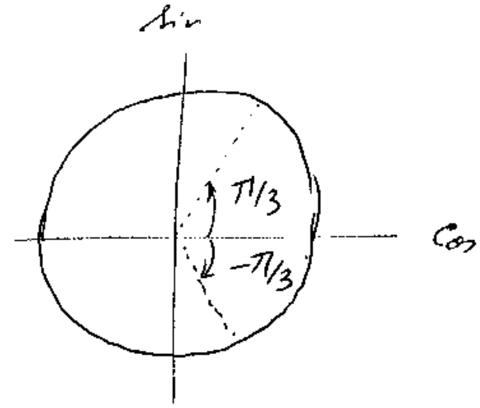
$$\frac{y'}{y} = \frac{1}{2x} + \frac{7}{x-5} + \frac{3}{x+6} - \frac{2}{x+1}$$

$$\Rightarrow y' = y\left(\frac{1}{2x} + \frac{7}{x-5} + \frac{3}{x+6} - \frac{2}{x+1}\right)$$

$$\Rightarrow y' = \frac{x^{1/2}(x-5)^7(x+6)^3}{(x+1)^2}\left(\frac{1}{2x} + \frac{7}{x-5} + \frac{3}{x+6} - \frac{2}{x+1}\right).$$

5.

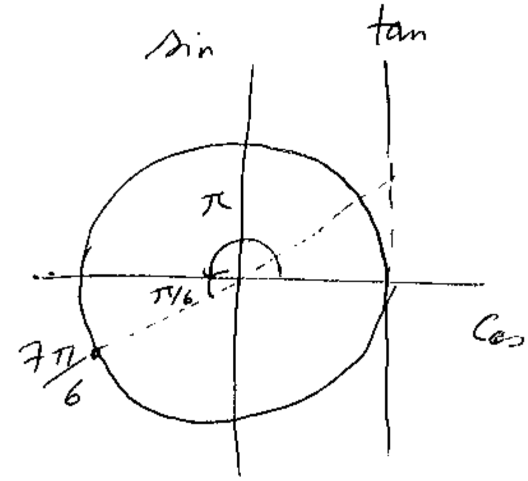
a) $\sin(-\pi/3) = -\sin \pi/3 = -\dfrac{\sqrt{3}}{2}$



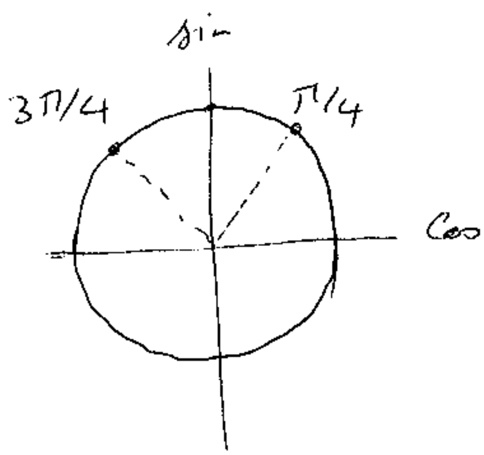
b) $\tan\left(\dfrac{7\pi}{6}\right)$

$= \tan\left(\pi + \dfrac{\pi}{6}\right) = \tan(\pi/6)$

$= \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$



c) $\sec(3\pi/4) = \dfrac{1}{\cos(3\pi/4)} = \dfrac{1}{\cos(\pi - \pi/4)} = \dfrac{1}{-\cos \pi/4}$



$= \dfrac{1}{-\dfrac{\sqrt{2}}{2}} = -\dfrac{2}{\sqrt{2}}$

$= -\dfrac{2\sqrt{2}}{\sqrt{2}\sqrt{2}} = -\sqrt{2}$

6.

$$\frac{\sec\theta}{\tan\theta+\cot\theta} = \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}} = \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}}$$

$$= \frac{\frac{1}{\cos\theta}}{\frac{1}{\cos\theta\sin\theta}} = \frac{\cos\theta\sin\theta}{\cos\theta} = \sin\theta.$$