Q1/sol'n:

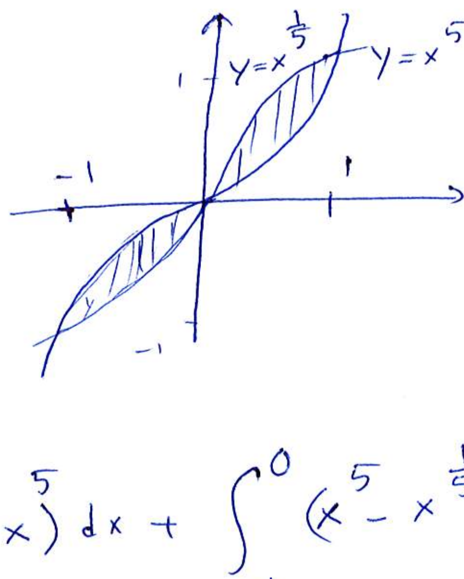
1st step: Finding two (or (maybe more)) points of intersections:

Setting algebraic equations of two curves equal to each other and solving for x.

$$x^5 = x^{\frac{1}{5}} \Longrightarrow (x^5)^5 = (x^{\frac{1}{5}})^5 \Longrightarrow$$

$$x^{25} = x \Longrightarrow x(x^{24}-1)=0 \Longrightarrow x = 0, \pm 1$$

2nd step:
area bounded by the two graphs $=$



$$\int_{-1}^{1} |f(x)-g(x)|\, dx =$$

$$\int_{-1}^{1} |x^{\frac{1}{5}} - x^5|\, dx = \int_0^1 (x^{\frac{1}{5}} - x^5)\, dx + \int_{-1}^{0} (x^5 - x^{\frac{1}{5}})\, dx =$$

$$\int_0^1 x^{\frac{1}{5}}\, dx - \int_0^1 x\, dx + \int_{-1}^{0} x^5\, dx - \int_{-1}^{0} x^{1/5}\, dx =$$

$$\frac{5}{6}x^{\frac{6}{5}}\Big|_0^1 - \frac{1}{6}x^6\Big|_0^1 + \frac{1}{6}x^6\Big|_{-1}^0 - \frac{5}{6}x^{\frac{6}{5}}\Big|_{-1}^0 = \boxed{\frac{4}{3}}$$

$$\left(\frac{5}{6}\right) - \left(\frac{1}{6}\right) + \left(-\frac{1}{6}\right) - \left(-\frac{5}{6}\right)$$

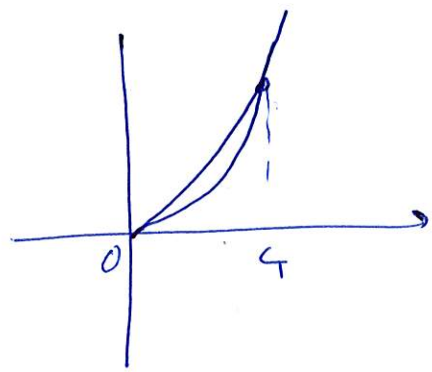
Q2 / sol'n :

1st step: Finding points of intersections:

$x^3 = \frac{x^3}{4} + 3x^2$

$x^3 - \frac{x^3}{4} = 3x^2$

$\frac{3}{4} x^3 = 3x^2$



$3x^2 \left(\frac{x}{4} - 1 \right) = 0 \quad \Rightarrow x = 0 \text{ or } x = 4$

2nd step: Evaluating the integral in order to find the volume of the given solid:

$\pi \int_a^b \left[f(x) - g(x) \right]^2 dx = \pi \int_0^4 \left[x^3 - \left(\frac{x^3}{4} + 3x^2 \right) \right]^2 dx =$

$\pi \int_0^4 \left(\frac{3}{4} x^3 - 3x^2 \right)^2 dx = \pi \int_0^4 \left(\frac{9}{16} x^6 + 9x^4 - 2 \cdot \frac{3}{4} x^3 \cdot 3x^2 \right) dx$

$= \pi \left(\frac{9}{16} \int_0^4 x^6 \, dx + 9 \int_0^4 x^4 \, dx - \frac{9}{2} \int_0^4 x^5 \, dx \right) =$

$= \pi \left(\frac{9 \cdot 4^4}{7} + \frac{9 \cdot 4^5}{5} - 3 \cdot 4^5 \right)$

Q3/sol'n:

a) $\int x^3 \ln x \, dx = ?$

Use integration by parts: $\begin{cases} u = \ln x \\ dv = x^3 dx \end{cases}$ $\begin{cases} du = \frac{dx}{x} \\ v = \frac{x^4}{4} \end{cases}$

$\left(\int u\,dv = uv - \int v\,du \right)$

$$\int x^3 \ln x \, dx = (\ln x) \frac{x^4}{4} - \int \frac{x^4}{4} \frac{dx}{x} =$$

$$(\ln x) \frac{x^4}{4} - \frac{1}{4} \frac{x^4}{4} + C = \boxed{\frac{1}{4} x^4 \left(\ln x - \frac{1}{4} \right) + C}$$

b) $\int x^2 \arctan x \, dx$

Again, it's a good idea to use integration by parts method: $\begin{cases} u = \arctan x \\ dv = x^2 dx \end{cases}$ $\begin{cases} du = \frac{dx}{1+x^2} \\ v = \frac{x^3}{3} \end{cases}$

$$\int x^2 \arctan x \, dx =$$

$$(\arctan x) \frac{x^3}{3} - \frac{1}{3} \int \frac{x^3}{1+x^2} dx =$$

(use partial fraction method) to compute this one!

$$\frac{x^3}{3} \arctan x - \frac{1}{6} x^2 + \frac{1}{6} \ln(x^2+1) + C$$

c) $\int_0^{\frac{\pi}{2}} x^2 \sin(3x)\, dx$

$$\begin{cases} u = x^2 \\ dv = \sin(3x)\,dx \end{cases} \qquad \begin{cases} du = 2x\,dx \\ v = \left(-\frac{1}{3}\right)\cos 3x \end{cases}$$

$$\int_0^{\frac{\pi}{2}} x^2 \sin(3x)\, dx = x^2\left(-\frac{1}{3}\right)\cos 3x \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \left(-\frac{1}{3}\right)\cos 3x\,(2x)\,dx$$

$$= -\frac{1}{3}x^2\cos 3x \Big|_0^{\frac{\pi}{2}} + \frac{2}{3}\int_0^{\frac{\pi}{2}} x\cos(3x)\,dx$$

using integration by parts again to find the value of this integral

$$= \left(-\frac{1}{3}\frac{\pi^2}{4}(0) - \left(-\frac{1}{3}\right)\frac{\pi^2}{4}(1)\right) + \text{whatever}$$

$$= \frac{\pi^2}{12} + \frac{2}{3}\int_0^{\pi/2} x\cos(3x)\,dx$$

$$= \frac{\pi^2}{12} + \frac{2}{3}\left(-\frac{\pi}{6} - \frac{1}{9}\right) = \ldots$$

$$\int_0^{\pi/2} x\cos(3x)\,dx =$$

$$\frac{x}{3}\sin(3x)\Big|_0^{\pi/2} - \frac{1}{3}\int_0^{\frac{\pi}{2}}\sin(3x)\,dx$$

$$= \frac{x}{3}\sin 3x\Big|_0^{\pi/2} + \frac{1}{3}\frac{1}{3}\cos 3x\Big|_0^{\pi/2}$$

$$= \left(\frac{\pi}{6}(-1) - 0\right) + \left(\frac{1}{9}(0) - \frac{1}{9}(1)\right) =$$

$$-\frac{\pi}{6} - \frac{1}{9}$$

Q4/sol'n:

a) First we decompose the fraction into subtraction of two simple fractions:

$$\frac{7x+1}{x^2-7x+12} = \frac{A}{x-3} + \frac{B}{x-4}$$

Now, we solve for A, and B:

$$A(x-4) + B(x-3) = 7x+1$$

$$(A+B)x + (-4A-3B) = 7x+1$$

$$\begin{cases} A+B=7 \\ -4A-3B=1 \end{cases} \Rightarrow A = -22, \; B = 29$$

$$\int \frac{7x+1}{x^2-7x+12} = \int \frac{-22}{x-3} + \int \frac{29}{x-4}\,dx =$$

$$-22 \ln|x-3| + 29 \ln|x-4| + C$$

(b) Notice that $\deg(x^3+5x+1)=3>2=\deg(x^2+5x)$

Therefore, we first divide numerator by denominator. $x^3+5x+1=(x-5)(x^2+5x)+30x+1$

Hence, $\dfrac{x^3+5x+1}{x^2+5x} = x-5+\dfrac{30x+1}{x^2+5x}$

which in turn implies

$$\int \frac{x^3+5x+1}{x^2+5x}\,dx = \int (x-5)\,dx + \int \frac{30x+1}{x^2+5x}\,dx =$$

$$\frac{1}{2}(x-5)^2 + \int\left(\frac{A}{x}+\frac{B}{x+5}\right)dx =$$

$$\frac{1}{2}(x-5)^2 + \frac{1}{5}\ln x + \frac{149}{5}\ln(x+5) + c$$

c) Note that you can't directly use method of partial fraction for trig functions.

So, we first substitute $\sin x$ by u.

Hence, $u = \sin x$ and $du = \cos x \, dx$

Thus, $\displaystyle\int \frac{\cos x}{\sin^2 x + 9\sin x + 20} \, dx = \int \frac{du}{u^2 + 9u + 20} =$

$$\int \frac{du}{(u+4)(u+5)} = \int \frac{A \, du}{u+4} + \int \frac{B \, du}{u+5} =$$

$$\int \frac{du}{u+4} + \int \frac{(-1) \, du}{u+5} = \ln|u+4| - \ln|u+5| + C$$

$$= \ln|\sin x + 4| - \ln|\sin x + 5| + C$$

d) $\int e^x \sin x \, dx$

We'll use the method of integration by parts twice in the course of solution.

First, let's use $\begin{cases} u = \sin x \\ dv = e^x dx \end{cases}$ $\begin{cases} du = \cos x \, dx \\ v = e^x \end{cases}$

Then,

$$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx \quad (I)$$

Now, we use the same method to compute $\int e^x \cos x \, dx$. Set $\begin{cases} u = \cos x \\ dv = e^x dx \end{cases}$ $\begin{cases} du = -\sin x \, dx \\ v = e^x \end{cases}$

Then $\int e^x \cos x = e^x \cos x - \int e^x (-\sin x) \, dx$

$$= e^x \cos x + \int e^x \sin x \, dx \quad (II)$$

From (I) and (II), we get

$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx =$

$e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

Therefor, $\boxed{\int e^x \sin x = \dfrac{e^x(\sin x - \cos x)}{2}}$

Q5 / sol'n :

(a) $\int_0^{\infty} \frac{1}{(5x+2)^6}\,dx$

using substitution $u = 5x+2$, we get

$$\int_0^{\infty} \frac{1}{(5x+2)^6}\,dx = \lim_{t\to\infty} \int_0^t \frac{1}{(5x+2)^6}\,dx = \frac{-1}{25} \lim_{t\to\infty} \left(\frac{1}{5x+2}\right)^5 \Big|_0^t$$

$$= \frac{-1}{25}\left(\frac{-1}{32}\right) = \frac{1}{800}$$

(b) $\int_{-5}^{\infty} x^2 e^{-5x^3}\,dx$

Set $u = -5x^3$. Then $du = -15x^2$

$$\int x^2 e^{-5x^3}\,dx = \frac{-1}{15}\int e^u\,du = \frac{-1}{15} e^u + C$$

$$= \frac{-1}{15} e^{-5x^3} + C$$

Now $\int_{-5}^{\infty} x^2 e^{-5x^3}\,dx = \lim_{t\to\infty} \frac{-1}{15} e^{-5x^3}\Big|_{-5}^t =$

$$\frac{-1}{15}\left(0 - e^{5^4}\right) = \frac{1}{15} e^{625}$$