Probability and NonLocal PDEs: Interplay and Cross-Impact (Swansea University, 17–19 September 2018)

Abstracts

Marton Balazs (Bristol)

Blocking measures, hills, and hydrodynamics

Abstract: 1-dimensional interacting particle systems have a well-known surface growth representation, so we decided to try them on modeling real-life surface evolution phenomena. The ones we picked were geomorphological processes that shape hill slopes in nature. The restriction that hills do not seem to grow indefinitely or vanish altogether quickly turned our attention to blocking measures of particle systems. I will explain how this idea arose, how one can use blocking measures to predict the shapes of hills, and what type of scaling fits naturally this scenario. I will also explain a bit about hydrodynamics around blocking measures, with special attention to the boundaries; this appears to be somewhat missing from the mathematics literature. (Joint work with Jacob Calvert, Partícia Gonçalves and Katerina Michaelides.)

Emeric Bouin (Paris Dauphine)

Logarithmic delays in non local Fisher–KPP problems

Abstract: We consider the non-local Fisher–KPP equation modeling a population with individuals competing with each other for resources with a strength related to their distance, and obtain the asymptotics for the position of the invasion front starting from a localized population. Depending on the behavior of the competition kernel at infinity, the location of the front is either as in the local case, or polynomial. This is a joint work with Christopher Henderson (U. Chicago) and Lenya Ryzhik (Stanford).

Elaine Crooks (Swansea)

Invasion speeds in a competition-diffusion model with mutation

Abstract: We consider a reaction-diffusion system modelling the growth, dispersal and mutation of two phenotypes. This model was proposed by Elliott and Cornell (2012), who presented evidence that for a class of dispersal and growth coefficients and a small mutation rate, the two phenotypes spread into the unstable extinction state at a single speed that is faster than either phenotype would spread in the absence of mutation. Using the fact that, under reasonable conditions on parameters, the spreading speed of the two phenotypes is linearly determinate, we show that the spreading speed is a non-increasing function of the mutation rate, determine the ratio at which the phenotypes occur in the leading edge in the limit of vanishing mutation, and discuss the effect of trade-offs between dispersal and growth on the spreading speed of the phenotypes. This talk is based on joint work with Luca Börger and Aled Morris (Swansea).

Raluca Eftimie (Dundee)

The impact of environmental stochasticity on a class of nonlocal hyperbolic models for self-organised biological aggregations

Abstract: The collective movement of animals occurs as a result of communication between the members of the community. However, inter-individual communication can be affected by the stochasticity of the environment, leading to changes in the perception of neighbours and subsequent changes in individual behaviour, which then influence the overall behaviour of the animal aggregations. To investigate the effect of noise on the overall behaviour of animal aggregations, we consider a class of nonlocal hyperbolic models for the collective movement of animals. We show that for some parameters the increase in noise can lead to a sequence of transitions between different spatio-temporal patterns, and these transitions are quite similar to the transitions obtained when we perturb deterministically these parameters.

Dmitri Finkelshtein (Swansea)

Stochastic dynamics of complex systems: mesoscopic description and beyond

Abstract: We discuss statistical description to the study of stochastic dynamics of complex systems in continuum. We give an overview of methods to show the existence and uniqueness of the corresponding evolutions of correlation functions, and consider mesoscopic (kinetic) description as a way to describe the quantitative and qualitative behavior of these functions. We consider examples of the appearing mesoscopic (kinetic) equations, which are nonlocal nonlinear PDE, and discuss variety of effects their solution have, and briefly the methods they require. Finally, we describe the method which allow us to find the next term of the approximation of real correlation functions beyond the mesoscopic description.

Carina Geldhauser (St Petersburg)

Optimizing the fractional power in a model with stochastic PDE constraints

Abstract: We study an optimization problem with SPDE constraints, which has the peculiarity that the control parameter s is the s-th power of the diffusion operator in the state equation. We discuss a suitable concept of solutions of the state equation and establish pathwise differentiability properties of the stochastic process w.r.t. the fractional parameter s. Finally, we show that under certain conditions on the noise, optimality conditions for the control problem can be established. This is joined work with Enrico Valdinoci (Milan).

John Herman (Warwick)

Extension Technique for Trace Processes: a Stochastic Approach

Abstract: It was proven by Caffarelli and Silvestre that for each $\alpha \in (0,2)$, the fractional Laplacian operator $-(-\Delta)^{\alpha/2}$ in $\mathbb{R}^d$ can be represented by the Dirichlet-to-Neumann map of a function $u_f : \mathbb{R}^d \times [0,\infty)$ satisfying a degenerate elliptic equation in the upper-half plane. This local characterisation of a non-local operator allows the use of PDE techniques to study equations involving the fractional Laplacian. In this talk, we discuss the probabilistic interpretation of the Caffarelli-Silvestre extension technique and present a generalisation of this representation using stochastic methods.

Ifan Johnston (Warwick)

Heat kernel estimates for fractional evolution equations

Abstract: In 1967 Aronson showed that the fundamental solution of the heat equation for a second order uniformly elliptic operator in divergence form satisfies two-sided Gaussian estimates. Using this celebrated result, we investigate two-sided estimates for the fundamental solution of the fractional analogues of the heat equation, where one replaces the time derivative with a Caputo fractional derivative of order $\beta \in (0,1)$ and also replace the second order elliptic operator with a homogeneous pseudo-differential operator. The starting point for these estimates is given by a formula, which is due to Zolotarev, that links Mittag–Leffler functions with β-stable densities via the Laplace transform. Probabilistically speaking, the solution of such fractional evolution equations is typically some time-changed Brownian motion, or time-changed stable process. This is joint work with Vassili Kolokoltsov.

Peter Kuchling (Bielefeld)

Glauber dynamics on the cone of discrete Radon measures

Abstract: We consider the Glauber dynamics on the cone of positive measures, i.e. a birth-and-death model emerging from a Dirichlet form defined via the underlying Gibbs measure. The particles are equipped with positive marks which influence the birth of new particles. The talk reviews the representation of particle configurations in the cone of discrete measures as well as some known results. Finally, we present an existence result for the dynamics on the cone and compare it to the unmarked case. (Joint work with Yuri Kondratiev and Dmitri Finkelshtein.)

Oleksandr Kutoviy (Bielefeld)

Nonlinear perturbations of evolution systems in scales of Banach spaces

Abstract: A variant of the abstract Cauchy–Kovalevskaya theorem is considered. We prove existence and uniqueness of classical solutions to the nonlinear, non-autonomous initial value problem

$$\frac{\mathrm{d}u(t)}{\mathrm{d}t} = A(t)u(t) + B(u(t), t), \quad u(0) = x$$

in a scale of Banach spaces. Here $A(t)$ is the generator of an evolution system acting in a scale of Banach spaces and $B(u, t)$ obeys an Ovcyannikov-type bound. Continuous dependence of the solution with respect to $A(t)$, $B(u, t)$ and x is proved. The results are applied to the Kimura–Maruyama equation for the mutation-selection balance model. This yields a new insight in the construction and uniqueness question for nonlinear Fokker-Planck equations related with interacting particle systems in the continuum.

Chris Lutsko (Bristol)

The Lorentz Gas: going beyond the Boltzmann–Grad limit

Abstract: We use coupling methods to prove a central limit theorem for the Lorentz process in a random scattering configuration in the low density limit as time is taken to infinity. It has long been known that the random Lorentz process, in the low-density limit, converges (in several senses) to a random flight process for a finite time. We use this connection to motivate coupling the two processes in order to prove a central limit theorem for the Lorentz process in infinite time. As Gallavotti showed that the Lorentz process obeys a linear Boltzmann equation in the finite time low-density limit, we hope this talk will indicate how probabilistic methods can be used to tackle a problem related to non-local PDE. This is joint work with Bálint Tóth.

Karsten Matthies (Bath)

Rescaled objective solutions of Fokker–Planck and Boltzmann equations

Abstract: We study the long-time behavior of symmetric solutions of the nonlinear Boltzmann equation and a closely related nonlinear Fokker–Planck equation.

Solutions are called objective if they symmetric with respect to some subgroup of the Euclidean symmetry group. We consider objective solutions where the symmetry corresponds to shear flows. The existence of stationary solutions can be ruled out because the energy is not conserved.

After anisotropic rescaling both equations conserve the energy. We show that the rescaled Boltzmann equation does not admit stationary densities of Maxwellian type (exponentially decaying). For the rescaled Fokker–Planck equation we demonstrate that all solutions converge to a Maxwellian in the long-time limit using hypocoercivity estimates, however the convergence rate is only algebraic, not exponential. Joint work with Florian Theil.

Sarah Penington (Bath)

Branching Brownian motion with decay of mass and the non-local Fisher–KPP equation

Abstract: We add a competitive interaction between nearby particles in a branching Brownian motion (BBM). Each particle has a mass, which decays at rate proportional to the mass density in a window centred at the location of the particle. The total mass of the system increases through branching events. In standard BBM, we may define the front location at time t as the location of the rightmost particle. For BBM with decay of mass, it makes sense to instead define the front location as the location at which the local mass density drops to $o(1)$. We can show that in a weak sense this front is $\sim t^{1/3}$ behind the front for standard BBM.

We can also show that at large times, over a bounded time interval, the local mass density for BBM with decay of mass is well approximated by the solution of a PDE known as the non-local Fisher–KPP equation. This relationship between the particle system and the PDE allows us to control the behaviour of the local mass density behind the front at large times, and also to use intuition from the particle system setting to prove results about the PDE.

Several interesting questions about this model remain open. Joint work with Louigi Addario-Berry and Julien Berestycki.

Mariya Ptashnyk (Heriot-Watt)

Homogenization of degenerate pseudoparabolic equations and variational inequalities

Abstract: Viscoelastic deformations of biological tissues and two-phase flow in a porous medium with dynamical capillary pressure can be modelled by pseudoparabolic-type equations and variational inequalities. In this talk we will consider microscopic model for viscoelastic deformations of a plant tissues taking into account its complex microstructure. An obstacle problem for unsaturated flow with dynamical capillary pressure in a perforated porous medium is modelled by degenerate pseudoparabolic variational inequality defined in a domain depending on a small parameter (the ration between the size of perforations and size of the whole domain).

Regularisation and penalty operator methods are applied to show the existence of a solution of the nonlinear degenerate pseudoparabolic variational inequality. The method of two-scale convergence is used to derive the macroscopic obstacle problem. Perturbation approach is applied to show a priori estimates for solutions of degenerate viscoelastic equations and to derive the corresponding macroscopic problem.

Pasha Tkachov (L'Aquila)

On stability of traveling wave solutions for integro-differential equations related to branching Markov processes

Abstract: The aim of this paper is to prove stability of traveling waves for integro-differential equations connected with branching Markov processes. In other words, the limiting law of the left-most particle of a (time-continuous) branching Markov process with a Lévy non-branching part is demonstrated. The key idea is to approximate the branching Markov process by a branching random walk and apply a result of Aïdékon on the limiting law of the latter one.

Lorenzo Toniazzi (Warwick)

Caputo evolution equations with time-nonlocal initial condition

Abstract: Consider the Caputo evolution equation (EE) $\partial_t^\beta u = \Delta u$ with initial condition ϕ on $\{0\} \times \mathbb{R}^d$. As it is well known, the solution reads $u(t,x) = \mathbf{E}_x[\phi(B_{E_t})]$. Here B_t is a Brownian motion and the independent time change E_t is an inverse β-stable subordinator. The fractional kinetic B_{E_t} is a popular model for subdiffusion, with remarkable universality properties.

We substitute the Caputo derivative ∂_t^β with the Marchaud derivative. This results in a natural extension of the Caputo EE featuring a *time-nonlocal initial condition* ϕ on $(-\infty, 0] \times \mathbb{R}^d$. We derive the new stochastic representation for the solution, namely $u(t,x) = \mathbf{E}_x[\phi(-W_t, B_{E_t})]$. This stochastic representation has a pleasing interpretation due to the non-obvious presence of W_t. Here W_t denotes the waiting/trapping time of the subdiffusion B_{E_t}. We discuss classical-wellposedness and time permitting weak-wellposedness for the respective extensions of Caputo-type EEs.