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Isoperimetric inequalities on \mathbb{R}^N with respect to homogeneous weights

Abstract

We solve a class of isoperimetric problems on \mathbb{R}^N with respect to monomial weights. For instance, we show that, if $k \in [0, 1]$, then among all smooth sets Ω in \mathbb{R}^N with fixed Lebesgue measure, $\int_{\partial\Omega} |x|^k dS$ achieves its minimum for a ball centered at the origin. Our results imply that the optimizers in some Caffarelli–Kohn–Nirenberg inequalities are radial. Further, they are used to obtain sharp apriori-bounds for solutions to some weighted elliptic BVPs.