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Topological branched coverings and invariant complex structures

Abstract

Let f be an orientation preserving branched covering of the two dimensional sphere. Is f realised (up to homotopy) by a rational function of the sphere? If yes, is the corresponding rational function unique up to Mobius transformations (the rigidity)? These questions amount to the existence and uniqueness of a complex structure that is invariant under the action of (the homotopy class of) f .

It turns out that the geometric and topological structure of “the orbits of the branched points” play a key role in these problems. When all the branched points are periodic, a classical result of W. Thurston provides a complete topological characterisation of the branched coverings that are realised by rational functions (and their uniqueness). On the other hand, when the orbits of the branched points form a more complicated subset of the sphere, say a Cantor set, the problem is highly nontrivial and has been extensively studied over the last three decades. In this talk we survey the main results of these studies, and describe a recent advance made on the uniqueness part using a renormalization technique.