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Long-time behaviour for monostable equations with non-local diffusion

Abstract

We consider a class of monostable non-linear equations with non-local diffusion in \mathbb{R}^d with bounded solutions. If the non-local diffusion is given through an anisotropic probability kernel decaying at least exponentially fast in a direction and if the initial condition has the similar property, then the solution propagates in this direction at most linearly in time. For a particular equation arising in population ecology, we study then travelling waves and a detailed description of the front of propagation. In contrast, for the general equation, if either the kernel or the initial condition have appropriately regular heavy tails, then an acceleration of the front propagation for solutions is observed. We present sharp estimates for the time-space zone which separates the region of convergence to the unstable zero solution with the region of convergence to the stable positive constant solution. We consider differences between the case when the initial condition decays in all directions at infinity with the case of the initial condition decreasing along all coordinate axes only. We obtain new estimates for solutions to a linear non-local equation as well.