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Representability and two-stage Postnikov systems

Abstract

Two-stage Postnikov systems classify spaces with only two consecutive non-vanishing homotopy groups. In the stable range, these are equivalent to spectra with only non-zero zeroth and first stable homotopy groups. But a variant of the fundamental groupoid gives another, equivalent algebraic characterization of such spectra: stable homotopy 1-types correspond to Picard categories. Classical constructions in K-theory will produce a stable 1-type from a Picard category, but in this talk I will discuss a kind of modified Brown representability using category-valued cohomology "groups" and the Yoneda lemma to directly produce a spectrum. This method, which I learned from Mikhail Kapranov, leads to some interesting algebraic questions about Picard categories; these are the topic of continuing joint work with Niles Johnson and Angelica Osorno.