LMS Undergraduate Summer School

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Colloquium

Caroline Series (Warwick)

Markov numbers and the free group on two generators

Abstract
Markov numbers are the integer solutions of the equation $x^2+y^2+z^2 = 3xyz$. Markov proved that the solutions can be arranged around the vertices of a trivalent tree, which leads to a tantalising unsolved problem called the uniqueness conjecture. Another well known classical theorem says that the possible sets of generators of the free group on two generators can be arranged in a similar way. It turns out that these two results are closely related. After explaining how all this works, we will go on to some modern work involving complex number solutions of the Markov equation where there are some very nice results but also many unanswered questions.