



# FRIEDEMANN BROCK

SYMMETRY, REARRANGEMENTS AND ISOPERIMETRY

## LEVERHULME TRUST LECTURES

DEPARTMENT OF MATHEMATICS, SWANSEA UNIVERSITY

7 AND 14 MARCH 2019

### Thursday 7 March, 3-5pm

LECTURE 1. AN INTRODUCTION TO REARRANGEMENTS

LECTURE 2. POLARISATION AND SYMMETRY OF SOLUTIONS TO VARIATIONAL PROBLEMS

### Thursday 14 March, 3-5pm

LECTURE 3. CONTINUOUS STEINER SYMMETRISATION AND SYMMETRY OF SOLUTIONS TO PDES

LECTURE 4. WEIGHTED REARRANGEMENTS AND ISOPERIMETRIC INEQUALITIES

THESE LECTURES ARE OPEN TO ALL AND THE MATERIAL WILL BE PRESENTED AT THE LEVEL ACCESSIBLE TO POSTGRADUATE STUDENTS. THE LECTURES WILL TAKE PLACE IN:

ROBERT RECORDE ROOM (CoFo 102), COMPUTATIONAL FOUNDRY  
SWANSEA UNIVERSITY BAY CAMPUS, FABIAN WAY, SWANSEA SA1 8EN

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Abertawe

# Lectures outline

## LECTURE 1. AN INTRODUCTION TO REARRANGEMENTS

WE SET UP THE GENERAL FRAMEWORK OF REARRANGEMENTS AND THEIR ROLE IN THE CALCULUS OF VARIATIONS. IN PARTICULAR, WE GIVE A LIST OF SYMMETRIZATIONS (SCHWARZ-, STEINER- AND SPHERICAL CAP SYMMETRIZATION) AND SOME INTEGRAL INEQUALITIES A LA HARDY-LITTLEWOOD AND POLYA-SZEGÖ.

## LECTURE 2. POLARISATION AND SYMMETRY OF SOLUTIONS TO VARIATIONAL PROBLEMS

WE INTRODUCE A SIMPLE TYPE OF REARRANGEMENT CALLED POLARIZATION. IT CAN BE USED TO GIVE ELEMENTARY PROOFS OF MANY INTEGRAL INEQUALITIES FOR SYMMETRIZATIONS. WE ALSO EXPLOIT POLARIZATION TO SHOW THAT THE MINIMISERS TO SOME CLASSICAL VARIATIONAL PROBLEMS ARE SYMMETRIC.

## LECTURE 3. CONTINUOUS STEINER SYMMETRISATION AND SYMMETRY OF SOLUTIONS TO PDES

WE INTRODUCE A HOMOTOPY OF REARRANGEMENTS CALLED CONTINUOUS STEINER SYMMETRIZATION. THE TOOL IS USED TO PROVE THAT NONNEGATIVE SOLUTIONS TO SOME DEGENERATE ELLIPTIC BOUNDARY VALUE PROBLEMS ARE SYMMETRIC. THESE SYMMETRY RESULTS GO BEYOND THE CLASSICAL ONES OBTAINED VIA THE TEXTBOOK MOVING PLANE METHOD.

## LECTURE 4. WEIGHTED REARRANGEMENTS AND ISOPERIMETRIC INEQUALITIES

WE DEVELOP APPLICATIONS OF WEIGHTED ISOPERIMETRIC INEQUALITIES AND RELATED SCHWARZ-TYPE REARRANGEMENTS TO FIND BEST CONSTANTS IN INEQUALITIES BETWEEN NORMS IN WEIGHTED FUNCTION SPACES, AS WELL AS SHARP A-PRIORI BOUNDS FOR SOLUTIONS TO SOME BOUNDARY VALUE PROBLEMS INVOLVING WEIGHTED ELLIPTIC OPERATORS.

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**PROFESSOR FRIEDEMANN BROCK** IS VISITING MATHEMATICS DEPARTMENT OF SWANSEA UNIVERSITY FROM SEPTEMBERS 2018 UNTIL AUGUST 2019. HIS VISITING PROFESSORSHIP HAS BEEN MADE POSSIBLE BY THE **LEVERHULME TRUST**.

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