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Invariance principle for non-homogeneous random walks with anomalous recurrence properties

Abstract

We consider a class of spatially non-homogeneous random walks in multidimensional Euclidean space with zero drift, which in any dimension (two or higher) can be recurrent or transient depending on the details of the walk. These walks satisfy an invariance principle, and have as their scaling limits a class of martingale diffusions, with law determined uniquely by an SDE with discontinuous coefficients at the origin. Furthermore, pathwise uniqueness of this SDE may fail. The radial coordinate of the diffusion is a Bessel process of dimension greater than 1. Unique characterization of the law of the diffusion, which must start at the origin, is natural via excursions built around the Bessel process; each excursion has a generalized skew-product-type structure, in which the angular component spins at infinite speed at the start and finish of each excursion. Defining appropriately the Riemannian metric g on the sphere S_{d-1} allows us to give an explicit construction of the angular component (and hence of the entire skew-product decomposition) as a time-changed Browninan motion with drift on the Riemannian manifold (S_{d-1}, g). In particular, this provides a multidimensional generalisation of the Pitman–Yor representation of the excursions of Bessel process with dimension between one and two. Furthermore, the density of the stationary law of the angular component with respect to the volume element of g can be characterised by a linear PDE involving the Laplace–Beltrami operator and the divergence under the metric g. This is joint work with Nicholas Georgiou and Andrew Wade.