

Fatemeh Mohammadi (Bristol)

A Combinatorial and Geometric View of System Reliability Theory

Abstract

Associated to every system/network there is a canonical ideal whose Hilbert series encodes the reliability of the system. We study various ideals arising in the theory of system reliability. Using ideas from the theory of orientations, and matroids on graphs we associate a polyhedral complex to our system so that the non-cancelling terms in the reliability formula can be read from the labeled faces of this complex. Algebraically, this polyhedron resolves the minimal free resolution of these ideals. In each case, we give an explicit combinatorial description of non-cancelling terms in terms of acyclic orientations of graph and the number of regions in the graphic hyperplane arrangement. This resolves open questions posed by Giglio–Wynn and develops new connections between the theory of oriented matroid, the theory of divisors on graphs, and the theory of system reliability.