Workshop

Pseudo-Differential Operators and Markov Processes

Marking 40 Years of Contributions to Mathematics by Professor Niels Jacob

Wednesday, 1 December & Thursday, 2 December 2021

Timetable

Wednesday, 1 December

10:00 - 10:15	Opening
10:15 - 10:50	Walter Hoh. *Niels Jacob. Mathematics and Life*
10:55 - 11:30	Erich Walter Farkas. *The mathematics of financial risk management: approach, challenges and recent developments*
11:30 - 11:55	*Virtual Tea/Coffee*
11:55 - 12:30	Victoria Knopova. *Recurrence and transience of some Lévy-type processes on* $\mathbb{R}$
Lunch Break	
14:15 - 14:50	Enrico Scalas. *Limit theorems for prices of options written on semi-Markov processes*
14:55 - 15:30	Franziska Kühn. *A Liouville theorem for Lévy generators, and applications in the martingale theory*
15:30 - 16:00	*Virtual Tea/Coffee*
16:00 - 16:35	Michael Röckner: *Strong dissipativity of generalized time-fractional derivatives and quasi-linear (stochastic) partial differential equations*
16:40 - 17:15	Lucian Beznea: *Scaling property for fragmentation processes related to avalanches*

Thursday, 2 December

09:30 - 10:05	Masayoshi Takeda. *Optimal Hardy-type Inequalities for Schrödinger Forms*
10:10 - 10:45	Tusheng Zhang. *Strong existence and uniqueness of solutions of SDEs with time dependent Kato class coefficients*
10:45 - 11:15	*Virtual Tea/Coffee*
11:15 - 11:50	Rudolf Hilfer: *Fractional glassy relaxation and convolution modules of distributions* The talk is CANCELLED
Lunch Break	
14:00 - 14:35	Eugene Shargorodsky. *On weak convergence of shift operators to zero on rearrangement-invariant spaces*
14:40 - 15:15	Krzysztof Bogdan. *New thoughts on potential theory in* L^p
15:15 - 15:45	*Virtual Tea/Coffee*
15:45 - 16:20	Dorothee Haroske. *Nuclear embeddings in function spaces – some recent results*
16:25 - 17:00	René Schilling. *Niels Jacob. Life and Mathematics*

Abstracts

Lucian Beznea, Institute of Mathematics, Bucharest
Title: **Scaling property for fragmentation processes related to avalanches**
Abstract: We emphasize a scaling property for the continuous time fragmentation processes related to a stochastic model for the fragmentation phase of an avalanche. We present numerical results that confirm the validity of the scaling property for our model, based on the appropriate stochastic differential equation of fragmentation and on a fractal property of the solution. The talk is based on joint works with Madalina Deaconu (Nancy) and Oana Lupaşcu-Stamate (Bucharest).

Krzysztof Bogdan, Wroclaw University of Science and Technology
Title: **New thoughts on potential theory in L^p**
Abstract: I will discuss recent and upcoming results on Markovian semigroups in the L^p setting and, time permitting, corresponding results for harmonic functions and integral forms.

Walter Farkas, University of Zurich & ETH Zurich
Title: **The mathematics of financial risk management: approach, challenges and recent developments**
Abstract: The risk of financial positions is measured by the minimum amount of capital to raise and invest in eligible portfolios of traded assets in order to meet a prescribed acceptability constraint. We discuss the interplay between analysis and probability theory in this area while addressing (some of) the current challenges.

Dorothee Haroske, Friedrich Schiller University of Jena
Title: **Nuclear embeddings in function spaces – some recent results**
Abstract: We study nuclear embeddings for spaces of Besov and Triebel-Lizorkin type. We concentrate on two special settings, where at least compactness is guaranteed: at first, some weighted setting, where the weight belongs to some Muckenhoupt class and is essentially of polynomial type. Secondly we deal with embeddings of spaces on quasi-bounded domains. Here we can extend our previous results on the compactness of corresponding embeddings. The concept of nuclearity goes back to Grothendieck (1955) and was the basis for many fundamental developments in functional analysis. Recently we noticed a refreshed interest to study such questions in special situations. This motivated our investigations. We obtain complete characterisations for the nuclearity of the corresponding embeddings in almost all cases. Essential tools are a discretisation in terms of wavelet bases, operator ideal techniques, as well as a very useful result of Tong (1969) about the nuclearity of diagonal operators acting in sequence spaces. This is joint work with Leszek Skrzypczak (Adam Mickiewicz University Poznań, Poland) and Hans-Gerd Leopold (Friedrich Schiller University Jena, Germany).

Rudolf Hilfer, University of Stuttgart
Title: **Fractional glassy relaxation and convolution modules of distributions**
Abstract: Solving fractional relaxation equations requires precisely characterized domains of definition for applications of fractional differential and integral operators. Determining these domains has been a longstanding problem. Applications in physics and engineering typically require extension from domains of functions to domains of distributions. In this work convolution modules are constructed for given sets of distributions that generate distributional convolution algebras. Convolutional inversion of fractional equations leads to a broad class of multinomial Mittag-Leffler type distributions. A comprehensive asymptotic analysis of these is carried out. Combined with the module construction the asymptotic analysis yields domains of distributions, that guarantee existence and uniqueness of solutions to fractional differential equations. The mathematical results are applied to anomalous dielectric relaxation in glasses. An analytic expression for the frequency dependent dielectric susceptibility is applied to broadband spectra of glycerol. This application reveals a temperature independent and universal dynamical scaling exponent. *Reference*: Analysis and Mathematical Physics (2021) volume 11, 130.
`https://doi.org/10.1007/s13324-021-00504-5`. This is joint work with T. Kleiner.

Victoria Knopova, Taras Shevchenko National University of Kyiv
Title: **Recurrence and transience of some Lévy-type processes on $\mathbb{R}$**
Abstract: This talk is about the transience, recurrence and ergodicity of a Lévy type process. I will briefly discuss the already known results, and present sufficient conditions for a Lévy-type process on $\mathbb{R}$, whose generator defined on the test functions is of the form

$$Lf(x) = \int_{\mathbb{R}} \big(f(x+u) - f(x) - \nabla f(x) \cdot u \mathbb{1}_{|u|\le 1}\big)\, \nu(x, du), \quad f \in C^2_\infty(\mathbb{R}).$$

Here $\nu(x, du)$ is a Lévy-type kernel, whose tails are either extended regularly varying or decay fast enough. The approach relies on the Foster-Lyapunov criterion.

Franziska Kühn, Technical University of Dresden
Title: **A Liouville theorem for Lévy generators, and applications in the martingale theory**
Abstract: The classical Liouville theorem states that any bounded solution u to the equation $\Delta u = 0$ is constant. More generally, any polynomially bounded solution u is a polynomial. In this talk, we present a generalization of this result to the wider class of integro-differential equations $Au = 0$, where A is the infinitesimal generator of a Lévy process. For the particular case of Brownian motion, the infinitesimal generator is $A = \frac{1}{2}\Delta$, and we recover the classical Liouville theorem. Moreover, we show that the Liouville theorem is closely related to a structural result for martingales associated to Lévy processes. Namely, if $(X_t)_{t\ge 0}$ is a "nice" Lévy process and f is a continuous, polynomially bounded function such that $M_t := f(X_t) - \mathbb{E}f(X_t)$ is a martingale, then f is a polynomial at most of order 2. The talk is based on a joint work with René Schilling.

Michael Röckner, University of Bielefeld & Academy of Mathematics and Systems Science, CAS, Beijing
Title: **Strong dissipativity of generalized time-fractional derivatives and quasi-linear (stochastic) partial differential equations**
Abstract: In this talk we shall identify generalized time-fractional derivatives as generators of C_0-operator semigroups and prove their strong dissipativity on Gelfand triples of properly in time weighted L^2-path spaces. In particular, the classical Caputo derivative is included as a special case. As a consequence one obtains the existence and uniqueness of solutions to evolution equations on Gelfand triples with generalized time-fractional derivatives. These equations are of type $\frac{d}{dt}(k * u)(t) + A(t, u(t)) = f(t)$, $0 < t < T$, with (in general nonlinear) operators $A(t, \cdot)$ satisfying general weak monotonicity conditions. Here k is a non-increasing locally Lebesgue-integrable nonnegative function on $[0, \infty)$ with $\lim_{s\to\infty} k(s) = 0$. Analogous results for the case, where f is replaced by a time-fractional additive noise, are obtained as well. Applications include generalized time-fractional quasi-linear (stochastic) partial differential equations. In particular, time-fractional (stochastic) porous medium and fast diffusion equations with ordinary or fractional Laplace operators or the time-fractional (stochastic) p-Laplace equation are covered.
Work supported by Deutsche Forschungsgemeinschaft (DFG) through "Collaborative Research Centre (CRC) 1283".' References: Liu/R./da Silva: J. Funct. Anal. 2021

Enrico Scalas, University of Sussex
Title: **Limit theorems for prices of options written on semi-Markov processes**
Abstract: We consider plain vanilla European options written on an underlying asset that follows a continuous time semi-Markov multiplicative process. We derive a formula and a renewal type equation for the martingale option price. In the case in which intertrade times follow the Mittag-Leffler distribution, under appropriate scaling, we prove that these option prices converge to the price of an option written on geometric Brownian motion time-changed with the inverse stable subordinator. For geometric Brownian motion time changed with an inverse subordinator, in the more general case when the subordinator's Laplace exponent is a special Bernstein function, we derive a time-fractional generalization of the equation of Black and Scholes. This is joint work with Bruno Toaldo. Link to the preprint: `https://arxiv.org/abs/2104.04817`

Eugene Shargorodsky, Kings College London
Title: **On weak convergence of shift operators to zero on rearrangement-invariant spaces**
Abstract: Let $\{h_n\}_{n\in\mathbb{N}}$ be a sequence in $\mathbb{R}^d$ tending to infinity and let $\{T_{h_n}\}$ be the corresponding sequence of shift operators given by $(T_{h_n}f)(x) = f(x-h_n)$ for $x \in \mathbb{R}^d$. We prove that $\{T_{h_n}\}$ converges weakly to the zero operator as $n \to \infty$ on a separable rearrangement-invariant Banach function space $X(\mathbb{R}^d)$ if and only if its fundamental function φ_X satisfies $\varphi_X(t)/t \to 0$ as $t \to \infty$. For a non-separable rearrangement-invariant Banach function space $X(\mathbb{R}^d)$, we show that $\{T_{h_n}\}$ does not converge weakly to the zero operator as $n \to \infty$ if (i) $h_n = nh,\ h \in \mathbb{R}^d \setminus \{0\}$, or (ii) $X(\mathbb{R}^d)$ is a Marcinkiewicz endpoint space $M_\varphi(\mathbb{R}^d)$ or an Orlicz space $L^\Phi(\mathbb{R}^d)$. This is a joint work with A. Karlovich.

Masayoshi Takeda, Tohoku University
Title: **Optimal Hardy-type Inequalities for Schrödinger Forms**
Abstract: For a subcritical Schrödinger form, we give a method to construct a critical Schrödinger form from the subcritical Schrödinger form by subtracting a suitable positive potential. The method enables us to obtain optimal Hardy-type inequalities.

Tusheng Zhang, University of Manchester
Title: **Strong existence and uniqueness of solutions of SDEs with time dependent Kato class coefficients**
Abstract: Consider stochastic differential equations (SDEs) in $\mathbf{R}^d$: $dX_t = dW_t + b(t, X_t)dt$, where W is a Brownian motion, $b(\cdot,\cdot)$ is a measurable vector field. It is known that if $|b|^2(\cdot,\cdot) = |b|^2(\cdot)$ belongs to the Kato class $\mathbf{K}_{d,2}$, then there is a weak solution to the SDE. In this article we show that if $|b|^2$ belongs to the Kato class $\mathbf{K}_{d,\alpha}$ for some $\alpha \in (0,2)$ (α can be arbitrarily close to 2), then there exists a unique strong solution to the stochastic differential equations, extending the results in the existing literature as demonstrated by examples. Furthermore, we allow the drift to be time-dependent. The new regularity estimates we established for the solutions of parabolic equations with Kato class coefficients play a crucial role.