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An Averaging Principle for Two-Time-Scale Functional Diffusions

Abstract

Delays are ubiquitous, pervasive, and entrenched in everyday life, thus taking it into consideration is necessary. Dupire recently developed a functional Itô formula, which has changed the landscape of the study of stochastic functional equations and encouraged a reconsideration of many problems and applications. Based on the new development, this work examines functional diffusions with two-time scales in which the slow-varying process includes path-dependent functionals and the fast-varying process is a rapidly-changing diffusion. The gene expression of biochemical reactions occurring in living cells in the introduction of this paper is such a motivating example. This paper establishes mixed functional Itô formulas and the corresponding martingale representation. Then it develops an averaging principle using weak convergence methods. By treating the fast-varying process as a random “noise”, under appropriate conditions, it is shown that the slow-varying process converges weakly to a stochastic functional differential equation whose coefficients are averages of that of the original slow-varying process with respect to the invariant measure of the fast-varying process.