Sublinear elliptic problems with a Hardy potential.

Catherine Bandle (Basel)

Abstract. Let $\Omega \subset \mathbb{R}^N$ be a bounded domain and $\delta(x)$ be the distance of a point $x \in \Omega$ to the boundary. We study the positive solutions of the problem $\Delta u + \frac{\mu}{\delta(x)^2} u = u^p$ in Ω, where $1 > p > 0$ and $\mu \in \mathbb{R}$, $\mu \neq 0$ is smaller then the Hardy constant. The interplay between the singular potential and the nonlinearity leads to interesting structures of the solution set. It turns out that there are essentially two types of positive solutions, those governed by the linear regime and those with a dead core caused by the nonlinearity. We give the complete picture of the radial solutions in a ball and study some extensions to general domains. The results are obtained in collaboration with V. Moroz, W. Reichel and A. Pozio.

[1] C. Bandle, V. Moroz and W. Reichel, *'Boundary blowup' type sub-solutions to semilinear elliptic equations with Hardy potential*, J. London Math. Soc. 77 (2008), 503-523.

[2] C. Bandle, M. A. Pozio, *Sublinear elliptic problems with a Hardy potential*, Nonlinear Analysis 119 (2015), 149-166.

[3] C. Bandle, M.A. Pozio, *Positive solutions of semilinear elliptic problems with a Hardy potential,* submitted for publication.