SCHRÖDINGER, POISSON, AND COMPACTNESS

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The nonlinear Schrödinger-Poisson system

$$\text{(NSP)} \quad \begin{cases} -\Delta u + \lambda u + \phi u = |u|^{p-1}u, & x \in \mathbb{R}^3, \\ -\Delta \phi = u^2, & x \in \mathbb{R}^3, \end{cases}$$

has been studied by several authors in relation to existence, nonexistence, multiplicity and behaviour of the solutions in the semi-classical limit, showing significant mathematical features which are not shared with nonlinear Schrödinger type equations. In the variational setting, the occurring lack of compactness phenomena are sensitive to both λ and p, and the scenario becomes interestingly rich when replacing the Laplacian with a fractional Laplacian and/or considering the effect of weight functions.
In my talk I will discuss some of these phenomena paying particular attention to decomposition properties, in the spirit of the celebrated Brezis-Lieb lemma. Reference will be made to the papers listed below.

References

[1] J. Bellazzini, M. Ghimenti, C. Mercuri, V. Moroz and J. Van Schaftingen. Sharp Gagliardo-Nirenberg inequalities in fractional Coulomb-Sobolev spaces, *to appear in Transactions of AMS* DOI: https://doi.org/10.1090/tran/7426

[2] D. Bonheure and C. Mercuri. Embedding theorems and existence results for nonlinear Schrödinger-Poisson systems with unbounded and vanishing potentials. *J. Differential Equations*, 2011, **251**, pp. 1056–1085.

[3] D. Bonheure, J. Di Cosmo and C. Mercuri. Concentration on circles for nonlinear Schrödinger-Poisson systems with unbounded potentials vanishing at infinity. *Commun. Contemp. Math.*, 2012, **14**(2), pp. 31.

[4] C. Mercuri, V. Moroz and J. Van Schaftingen. Groundstates and radial solutions to nonlinear Schrödinger-Poisson-Slater equations at the critical frequency. *Calc. Var. Partial Differential Equations*, 2016, **55**(6), pp. 1– 58.

[5] C. Mercuri and M. T. Tyler. On a class of nonlinear Schrödinger-Poisson systems involving a nonradial charge density *In preparation.*

[6] C. Mercuri and M. Willem. A global compactness result for the p-Laplacian involving critical nonlinearities. *Discrete and Continuous Dynamical Systems*, 2010, **28**(2), pp. 469–493.

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