Stochastic homogenisation for Hamilton-Jacobi equations.

In the first part of the talk I will present a result from 1999 by Souganidis and Rezakhanlou-Tarver on stochastic homogenisation for equations of the form

$$u_t^\varepsilon + H\left(\frac{x}{\varepsilon}, Du, \omega\right) = 0, \quad x \in R^N$$

where H satisfies standard regularity assumptions ($C_1^{-1}(|p|^\alpha - 1) \leq H(x,p,\omega) \leq C_1(1+|p|^\alpha)$ with $\alpha > 1$ and $|H(x,p,\omega) - H(y,p,\omega)| \leq m(|x-y|(1+|p|))$ with $m(t)$ modulus of continuity) and ω is a random variable in a probability space $\big(\Omega, \mathcal{F}, P\big)$ and with initial condition $u_0 \in BUC(\mathbb{R}^n)$.

If the Hamiltonian is stationary ergodic, then it is possible to show that the effective problem is deterministic. More precisely one can prove that the viscosity solutions of the stochastic solutions $u^\varepsilon(t,x,\omega)$ uniformly converge to a determinist function $u(t,x)$ which can be characterised as the viscosity solution for an equation of the form $u_t + \overline{H}(Du) = 0$ (with the same initial condition u_0). Stationary ergodicity is the key ingredient to get a deterministic limit problem.

The main idea in both the papers is to use variational formulas for the solution of the ε-problems and restrict somehow the attention to affine initial conditions.

In the second part of the talk I will present an ongoing project with Paola Mannucci and Claudio Marchi (both from University of Padova) to extend the result to non-coercive Hamiltonians. In particular we will focus on Hamiltonians of the form $H(\text{"}x/\varepsilon\text{"}, \sigma(x)Du, \omega)$, where $\sigma(x)$ which is a $m \times n$ matrix of Carnot-type. I will explain how it is convenient to deal with these more degenerate Hamiltonians by interpreting them as PDEs in a Carnot group (i.e. modifying the geometric/metric structure of the underlying space). Carnot groups are anisotropic at any scale, then the scale "x/ε" will need to be adapted to the new group structure defined on $\mathbb{R}^n$. Many difficulties occur when dealing with such a degenerate underlying structure (which reflects the non-coercive dependence on the gradient) especially because the idea of approximation by affine functions fails.