Second South West Regional PDE Winter School

15-16 January 2010

Department of Mathematics, Swansea University

Organised by:

- Oxford Centre for Nonlinear PDE
- University of Bath
- University of Bristol
- The University of Warwick
- Wales Institute of Mathematical and Computer Sciences
- Department of Mathematics, Swansea University

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2 Sponsors

- Oxford Centre for Nonlinear PDE
- Wales Institute of Mathematical and Computational Sciences

3 Programme

	Friday 15 January
12:00-12:50	Arrivals Lunch in Mathematics Reading Room (room 213)
12:50-13:00	Opening of the meeting
13:00-14:20	**Nina Uraltseva** (St. Peterburg, Russia) *Maximum principle and related topics, I*
14:30-15:30	**Jean Van Schaftingen** (Luovain-La-Neuve, Belgium) *Limiting estimates for L^1 vector fields*
15:30-16:00	Tea
16:00-17:20	**Giuseppe Mingione** (Parma, Italy) *Non-Linear aspects of Calderón-Zygmund theory, I*
17:30-18:30	*Short Presentations*
19:00	Reception and Dinner at Fulton House
	Saturday 16 January
9:00-10:30	**Giuseppe Mingione** (Parma, Italy) *Non-Linear aspects of Calderón-Zygmund theory, II*
10:30-11:00	Coffee
11:00-12:40	*Short Presentations*
13:00-14:00	Lunch in Mathematics Reading Room 213
14:00-15:30	**Nina Uraltseva** (St. Peterburg, Russia) *Maximum principle and related topics, II*
15:30-16:00	Tea
16:00-17:00	**Giuseppe Buttazzo** (Pisa, Italy) *On some problems in spectral optimization*

All Lectures and Short Presentations will be held in the Mathematics Seminar Room (room 224 in the Talbot Building, labelled 8.3 on the campus map). Coffee and tea will be served in the adjacent Mathematics Common Room.

4 Minicourses

- Giuseppe Mingione (Parma, Italy)
  Non-linear aspects of Calderón–Zygmund theory.

  Synopsis. Calderón–Zygmund theory is a classical tool that allows, in the case of linear problems, to derive sharp integrability properties of solutions starting from those of the given data. I will try to give an account of certain aspects which remain valid for non-linear equations.

- Nina Uraltseva (St Petersburg, Russia)
  Maximum principle and related topics.

  Abstract. Strong maximum principle is an important tool in the theory of elliptic and parabolic equations of the second order. Qualitative properties of solutions such as the Harnack inequality, the Liouville theorem, boundary gradient estimates and the Hopf lemma are close related to maximum principle. The goal of the lectures is to investigate these properties under the optimal conditions on the coefficients of equations and to present some applications of the results.

5 Lectures

- Giuseppe Buttazzo (Pisa, Italy)
  On some problems in spectral optimization.

  Abstract. We deal with shape optimization problems where the cost is a function of the eigenvalues of the Laplace operator with Dirichlet conditions on the free boundary. We show a general existence result for optimal domains under various kinds of constraints on the class of admissible choices and discuss some open questions.

- Jean Van Schaftingen (Louvain-la-Neuve, Belgium)
  Limiting estimates for L^1 vector fields

  Abstract. I will start from a recent work of Bourgain and Brezis which shows that one can recover some limiting estimates for problems of the form $-\Delta u = f$, where $f \in \mathrm{L}^1(\mathbf{R}^n, \mathbf{R}^n)$ is a divergence-free vector field. I shall present these results, their implications for Hodge decomposition, their connection with the isoperimetric inequality and an elementary method of proof; I shall compare them with existing tools such as real Hardy spaces; and I shall discuss the extensions to various settings as higher-order conditions, fractional and Lorentz spaces, estimates on domains and manifolds, and vector fields on the Heisenberg group.

6 Short Presentations

Friday 15 January: 5.30pm - 6.30pm

5.30pm: **Benjamin Jaye** (Missouri, USA)

Positive solutions of nonlinear equations with natural growth terms.

Abstract. We will present conditions for solvability along with global bounds for positive solutions of quasilinear and fully nonlinear operators perturbed by a 'natural growth' term. The model equations for our study are:

$$-\Delta_p u = \sigma |u|^{p-2} u + \omega$$

and

$$F_k(-u) = \sigma u^k + \omega$$

where σ and ω are nonnegative Borel measures. Here Δ_p is the quasilinear p-Laplacian operator, defined by $\Delta_p u = \mathrm{div}(|\nabla \mathrm{u}|^{\mathrm{p}-2}\nabla \mathrm{u})$ and $F_k(u)$ is the fully nonlinear k-Hessian operator, defined by

$$F_k(u) = \sum_{1 \le i_1 < \cdots < i_k \le n} \lambda_{i_1} \dots \lambda_{i_k}$$

where $\lambda_1, \dots \lambda_n$ are the eigenvalues of the Hessian matrix of u. The results presented are joint work with Igor E. Verbitsky.

5.50pm: **Giovanni Franzina** (Padua, Italy)

On a more general nonlinear eigenvalue problem.

Abstract. Let $N \in \mathbb{N}$, $p \in (1, +\infty)$, $q \in (1, p^*)$, where $p^* = Np/(N-p)$ if $p < N$ and $p^* = +\infty$ if $p \ge N$. We consider the quasilinear PDE

$$-\mathrm{div}(|\nabla \mathrm{u}|^{\mathrm{p}-2}\nabla \mathrm{u}) = \lambda \|\mathrm{u}\|_{\mathrm{L}^{\mathrm{q}}(\Omega)}^{\mathrm{p}-\mathrm{q}} |\mathrm{u}|^{\mathrm{q}-2}\mathrm{u}, \qquad \text{(P)}$$

in a domain of finite Lebesgue measure in $\mathbb{R}^N$, with the Dirichlet conditions $u = 0$ on the boundary $\partial\Omega$. We will point out that several results known for the case $q = p$, when (P) reduces to the typical eigenvalue problem for the p-laplacian, extend to the more general case $q \in (1, p^*)$. It turns out that (P) can be regarded as well as an eigenvalue problem. In particular, we will focus on surveying some uniqueness results concerning the first eigenvalue

$$\lambda_1(p, q) = \min_{0 \neq u \in W_0^{1,p}(\Omega)} \frac{\int_\Omega |\nabla u|^p dx}{(\int_\Omega |u|^q dx)^{p/q}}.$$

In the "subtypical" cases $q \le p$ the first eigenvalue is simple for all domain Ω of finite Lebesgue measure, whereas, if $q > p$, it this false in general. Nevertheless, if Ω is a ball in $\mathbb{R}^N$, then $\lambda_1(p, q)$ is simple for all $q \in (1, p^*)$.

6.10pm: **Mikhail Surnachev** (Swansea, UK)

A Harnack Inequality for Weighted Degenerate Parabolic Equations.

Abstract. I generalize the recent result of DiBenedetto, Gianazza, Vespri on the Harnack inequality for degenerate parabolic equations to the case of a weighted p-Laplacian type operator in the spatial part. The weight is assumed to belong to the suitable Muckenhoupt class.

Saturday 16 January: 11:00am - 12:40pm

11:00am: **Michael Helmers** (Oxford, UK)

Towards kinks in membranes with lateral phase separation.

Abstract. In the spontaneous curvature model for two-component lipid bilayers equilibrium shapes are described as closed surfaces minimizing the bending energy of each component plus an interface contribution. Although this energy does not enforce smoothness across interfaces, kinks are usually not included in the analysis. In the talk we consider the above model for rotationally symmetric membranes and a one-dimensional analogue when kinks are not excluded a priori. We introduce a family of energies for smooth curves and phase fields, and we study its sharp interface limit.

11:20am: **Ben Lambert** (Durham, UK)

Long Term Existence of Non-Parametric Mean Curvature Flow with a Neumann Boundary Condition.

Abstract. Following Huiskens 1989 paper on a boundary problem for Mean Curvature Flow, I will prove a gradient bound on a graph over a general compact domain with a perpendicular Neumann boundary condition. This in turn shows the uniform parabolicity of the evolution equation for the graph and so leads to a Long Term Existence Theorem. With a little extra work we see that we in fact have convergence to a constant function.

11:40am: **Sergey Morozov** (UCL, UK)

Lower Bound on the Density of States for Periodic Schrödinger Operators.

Abstract. We consider Schrödinger operators in R^d ($d > 2$) with smooth periodic potentials and prove a uniform lower bound on the density of states for large values of spectral parameter.

12:00pm: **Filip Rindler** (Oxford, UK)

Characterization of generalized gradient Young measures in $W^{1,1}$ and BV.

Abstract. Generalized Young measures (or DiPerna/Majda measures) are an extension of classical Young measures that are able to quantitatively account for both oscillation and concentration phenomena in merely L^1-bounded sequences of functions or in a sequence of Radon measures (uniformly bounded in the total variation norm). After explaining the basic framework in a functional-analytic spirit, the main result of this work is presented: A characterization of the class of generalized Young measures that can be generated by $W^{1,1}$-gradients or BV-derivatives. This characterization is in terms of Jensen-type inequalities for quasiconvex functions and can be considered a natural generalization of the Kinderlehrer-Pedregal Theorem for classical Young measures. This is joint work with Jan Kristensen.

12:20pm: **Path Soneji** (Oxford, UK)

Lower Semicontinuity in BV of quasiconvex integrands of subquadratic growth.

Abstract. A lower semicontinuity result in BV is obtained for quasiconvex integrals of subquadratic growth, defined on generalised Orlicz-Sobolev Spaces. The key step in this proof involves adapting techniques of Meyers and Malý to obtain some boundedness properties for an extension operator. A similar result is obtained by Kristensen in, where there are weaker assumptions on convergence but the integral needs to satisfy a stronger growth condition.

7 Participants

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