LIMITING ESTIMATES FOR L^1 VECTOR FIELDS

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Abstract. I will start from a recent work of Bourgain and Brezis which shows that one can recover some limiting estimates for problems of the form $-\Delta u = f$, where $f \in \mathrm{L}^1(\mathbf{R}^n, \mathbf{R}^n)$ is a divergence-free vector field. I shall present these results, their implications for Hodge decomposition, their connection with the isoperimetric inequality and an elementary method of proof; I shall compare them with existing tools such as real Hardy spaces; and I shall discuss the extensions to various settings as higher-order conditions, fractional and Lorentz spaces, estimates on domains and manifolds, and vector fields on the Heisenberg group.