Homogenisation of elastic composite plates in the nonlinear bending regime

Cherdantsev M.

Cardiff University

We consider a problem of homogenisation (i.e. finding an effective limit description) of a thin elastic periodic composite plate in the bending regime as both parameters - thickness of the plate h and period of the composite microstructure ε - go to zero. The plate in the reference configuration occupies thin domain $\Omega_h := \omega \times [-h/2, h/2]$, where $\omega \subset \mathbb{R}^2$, $h << 1$. Our setting is fully non-linear, the elastic energy of the deformation $u \in H^1(\Omega_h)$ is given by

$$\int_{\Omega_h} W(\varepsilon^{-1}x, \nabla u)dx,$$

where $W(y, \xi)$ is the stored elastic energy function periodic with respect to the in-plane variable $y \in \mathbb{R}^2$. In the non-linear bending regime (which allows displacements of order one, not to be confused with the Föppl - von Karman bending theory) the elastic energy is of order h^3. The limit homogenised elastic functional (after the appropriate rescaling) is given by the formula

$$\int_{\omega} Q_{hom}(\mathrm{II}(x_1, x_2))dx_1dx_2,$$

defined on the second fundamental form II of (roughly speaking) the mid-surface of the bending deformation u. However, the formula for the quadratic form $Q_{hom}(\mathrm{II})$ is different for different ε-to-h ratios. In particular, in the regime when $h << \varepsilon^2$ we have obtained very interesting and somewhat surprising result implying that the homogenised stored elastic energy function $Q_{hom}(\mathrm{II})$ is an everywhere discontinuous function of its argument II.