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Wasserstein geometry and Ricci curvature bounds for Poisson spaces

Abstract
Let Y be the configuration space over a complete and separable metric base space, endowed with the Poisson measure π. We study the geometry of Y from the point of view of optimal transport and Ricci-lower bounds. To do so, we define a formal Riemannian structure on $P_1(Y)$, the space of probability measures over Y with finite first moment, and we construct an extended distance W on $P_1(Y)$. The distance W corresponds, in our setting, to the Benamou–Brenier variational formulation of the Wasserstein distance. Our main technical tool is a non-local continuity equation defined via the difference operator on the Poisson space. We show that the closure of the domain of the relative entropy is a complete geodesic space, when endowed with W. We establish non-local infinite-dimensional analogues of results regarding the geometry of the Wasserstein space over a metric measure space with synthetic Ricci curvature bounded below. In particular, we obtain that: (a) the Ornstein–Uhlenbeck semi-group is the gradient flow of the relative entropy; (b) the Poisson space has Ricci curvature bounded below by 1 in the entropic sense; (c) the distance W satisfies an HWI inequality.

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