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MINI-COURSE

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Geometry of nets of conics

Abstract

A quadratic polynomial in two variables defines a curve in a plane known as conic. Over complex numbers, one can use projective transformations to classify such curves into three types: non-degenerate conic (corresponding to real conics: ellipse, hyperbola and parabola) and two singular conics (union of two distinct lines and a double line). A similar classification problem can be considered for a pencil of conics defined by a linear combination of two quadratic polynomials. This problem has been solved in 19 century by Corrado Segre (in much more general setting) and its solution is given by the types of singular conics in the pencil. In this course, we will explain how to use projective transformations to classify nets of conics (vector spaces spanned by three linearly independent quadratic polynomials in two variables). This classification has been done in 1977 by Terry Wall, and independently by Anthony Iarrobino and Jacques Emsalem in 1974 (unpublished). This problem is closely related to the classification of (possibly singular) cubic curves in the plane. We will compare two approaches to the classification and consider some applications in three-dimensional geometry.