LMS Undergraduate Summer School
Swansea, 12-23 July 2021
Programme

Monday, 12 July

09.45–10.00 Opening of the School

10.00–11.00 Sarah Whitehouse: Mini-course "Combinatorics of Young tableaux and symmetric groups," Lecture 1

11.00–11.30 Virtual Tea/Coffee Break

11.30–12.30 Sophie Morier-Genoud: Mini-course "Arithmetic and combinatorics of Conway–Coxeter frieze patterns," Lecture 1

12.30–14.30 Lunch & Free Time

14.30–15.30 Sarah Whitehouse: Mini-course "Combinatorics of Young tableaux and symmetric groups," Lecture 2

15.30–16.00 Virtual Tea/Coffee Break

16.00–17.30 Caroline Series: Colloquium "Markov numbers and the free group on two generators"

Tuesday, 13 July

10.00–11.00 Sarah Whitehouse: Mini-course "Combinatorics of Young tableaux and symmetric groups," Lecture 3

11.00–11.30 Virtual Tea/Coffee Break

11.30–12.30 Sophie Morier-Genoud: Mini-course "Arithmetic and combinatorics of Conway–Coxeter frieze patterns," Lecture 2

12.30–14.30 Lunch & Free Time

14.30–15.30 Sarah Whitehouse: Mini-course "Combinatorics of Young tableaux and symmetric groups," Lecture 4

15.30–16.00 Virtual Tea/Coffee Break

16.00–17.30 Felipe Rincon: Colloquium "A glimpse of tropical geometry"

Wednesday, 14 July

10.00–11.00 Sarah Whitehouse: Mini-course "Combinatorics of Young tableaux and symmetric groups," Lecture 5

11.00–11.30 Virtual Tea/Coffee Break

11.30–12.30 Ivan Cheltsov: Mini-course "Geometry of nets of conics," Lecture 1

12.30–14.30 Lunch & Free Time

14.30–15.30 Ivan Cheltsov: Mini-course "Geometry of nets of conics," Lecture 2

15.30–16.00 Virtual Tea/Coffee Break

16.00–17.30 Christina Cobbold: Colloquium "Mathematical modelling tools for vector borne diseases"

Thursday, 15 July

10.00–11.00 Sophie Morier-Genoud: Mini-course "Arithmetic and combinatorics of Conway–Coxeter frieze patterns," Lecture 3

11.00–11.30 Virtual Tea/Coffee Break

11.30–12.30 Sophie Morier-Genoud: Mini-course "Arithmetic and combinatorics of Conway–Coxeter frieze patterns," Lecture 4

12.30–14.30 Lunch & Free Time

14.30–15.30 Ivan Cheltsov: Mini-course "Geometry of nets of conics," Lecture 3

15.30–16.00 Virtual Tea/Coffee Break

16.00–17.00 Ivan Cheltsov: Mini-course "Geometry of nets of conics," Lecture 4

Friday, 16 July

10.00–11.00 Ivan Cheltsov: Mini-course "Geometry of nets of conics," Lecture 5

11.00–11.30 Virtual Tea/Coffee Break

11.30–12.30 Sophie Morier-Genoud: Mini-course "Arithmetic and combinatorics of Conway–Coxeter frieze patterns," Lecture 5

12.30–14.30 Lunch & Free Time

14.30–16.00 Anna Felikson: Colloquium "Quiver mutations and triangulated surfaces"

Monday, 19 July

10.00–11.00 Enrico Scalas: Mini-course "Monte Carlo simulations of anomalous random walks," Lecture 1

11.00–11.30 Virtual Tea/Coffee Break

11.30–12.30 Carlo Marinelli: Mini-course "Analytic inequalities," Lecture 1

12.30–14.30 Lunch & Free Time

14.30–15.30 Enrico Scalas: Mini-course "Monte Carlo simulations of anomalous random walks," Lecture 2

15.30–16.00 Virtual Tea/Coffee Break

16.00–17.30 Ian Leary: Colloquium "Countability, groups, subgroups and finiteness properties"

Tuesday, 20 July

10.00–11.00 Enrico Scalas: Mini-course "Monte Carlo simulations of anomalous random walks," Lecture 3

11.00–11.30 Virtual Tea/Coffee Break

11.30–12.30 Carlo Marinelli: Mini-course "Analytic inequalities," Lecture 2

12.30–14.30 Lunch & Free Time

14.30–15.30 Enrico Scalas: Mini-course "Monte Carlo simulations of anomalous random walks," Lecture 4

15.30–16.00 Virtual Tea/Coffee Break

16.00–17.30 Michael Grinfeld: Colloquium "Coagulation–fragmentation modelling"

Wednesday, 21 July

10.00–11.00 **Enrico Scalas**: Mini-course "Monte Carlo simulations of anomalous random walks," Lecture 5

11.00–11.30 **Virtual Tea/Coffee Break**

11.30–12.30 **Carlo Marinelli**: Mini-course "Analytic inequalities," Lecture 3

12.30–14.30 **Lunch & Free Time**

14.30–15.30 **Tobias Kuna**: Mini-course "Can one see the forest without loosing sight of the trees?" Lecture 1

15.30–16.00 **Virtual Tea/Coffee Break**

16.00–17.30 **Arkadiusz Wiśniowski**: Colloquium "Measuring migration by using traditional and new forms of data"

Thursday, 22 July

10.00–11.00 **Tobias Kuna**: Mini-course "Can one see the forest without loosing sight of the trees?" Lecture 2

11.00–11.30 **Virtual Tea/Coffee Break**

11.30–12.30 **Carlo Marinelli**: Mini-course "Analytic inequalities," Lecture 4

12.30–14.30 **Lunch & Free Time**

14.30–15.30 **Tobias Kuna**: Mini-course "Can one see the forest without loosing sight of the trees?" Lecture 3

15.30–16.00 **Virtual Tea/Coffee Break**

16.00–17.00 **Tobias Kuna**: Mini-course "Can one see the forest without loosing sight of the trees?" Lecture 4

Friday, 23 July

10.00–11.00 **Tobias Kuna**: Mini-course "Can one see the forest without loosing sight of the trees?" Lecture 5

11.00–11.30 **Virtual Tea/Coffee Break**

11.30–12.30 **Carlo Marinelli**: Mini-course "Analytic inequalities," Lecture 5

12.30–14.30 **Lunch & Free Time**

14.30–16.00 **Melanie Rupflin**: Colloquium "Asymptotic behaviour of gradient flows"

Abstracts

Mini-courses

Ivan Cheltsov (University of Edinburgh): Geometry of nets of conics

Abstract. A quadratic polynomial in two variables defines a curve in a plane known as conic. Over complex numbers, one can use projective transformations to classify such curves into three types: non-degenerate conic (corresponding to real conics: ellipse, hyperbola and parabola) and two singular conics (union of two distinct lines and a double line). A similar classification problem can be considered for a pencil of conics defined by a linear combination of two quadratic polynomials. This problem has been solved in 19 century by Corrado Segre (in much more general setting) and its solution is given by the types of singular conics in the pencil. In this course, we will explain how to use projective transformations to classify nets of conics (vector spaces spanned by three linearly independent quadratic polynomials in two variables). This classification has been done in 1977 by Terry Wall, and independently by Anthony Iarrobino and Jacques Emsalem in 1974 (unpublished). This problem is closely related to the classification of (possibly singular) cubic curves in the plane. We will compare two approaches to the classification and consider some applications in three-dimensional geometry.

Tobias Kuna (University of Reading): Can one see the forest without loosing sight of the trees?

Abstract. As in the above cited proverb, it can be very challenging to see the essential among too much information or detail available. This situation appears in a lot of areas: molecules in a glass of water, stars in a galaxy, blade of grass in a field, snakes in pit, any kind of data classification. In order to get a handle on the problem, a typical approach is to compute a few numbers, the so-called characteristics, and the hope is that these characteristics capture important information about the system. Think for example of a liquid: we know its density, we know its colour, we may X-ray it and so on. What do these characteristics tell us about the molecules building up the liquid? In other contexts though, it may not be so clear what are useful characteristics to be considered (e.g. the mean, the median, correlations, clusters etc.) It is an art to develop good characteristics for the problem under study. A myriad of characteristics have and are developed in this way.

In these lectures we will take a step back and take a conceptual view on the problem. What will a particular collection of characteristics tell us about our system? Assume the only information we have about our system is the values of a few characteristics, what can we conclude about our system, or in other words, which information do the characteristics contain?

In order to be a bit more concrete, we have to formulate what we mean by a general system. In the lectures, we will concentrate on systems which are given as a set on which we have a probability. For examples, liquids are described by probability distributions on the space of all possible positions of the molecules. We will consider easier sets in the lectures, like for example the natural numbers.

Let me give a concrete and simple example. Consider a probability on the real numbers (that is a "random" number). Assume we know the mean and the variance of the probability but nothing else about the probability distribution. We have the suspicion that our system is discrete or in other words quantised. Can that be or do our characteristics rule that out? What can you say about other restrictions? What happens if you choose a pair of numbers?

These may seem simple questions, but they are very challenging, have a long history and go under the name of "Moment Problem." Moment problems were among the motivating examples for the development of modern probability, functional analysis and convex analysis. There are interesting connections with algebra geometry and number theory. Quickly one reaches unsolved problems. The lectures will not require any pre-knowledge of probability or functional analysis, but they require an enthusiasm for real analysis and a love for constructing mathematical objects and a passion for solving puzzles.

Carlo Marinelli (UCL): Analytic inequalities

Abstract. Starting from the simple arithmetic mean–geometric mean inequality, we shall discuss inequalities of Cauchy–Schwarz and Hölder type in increasingly general settings, culminating with the proof of a (case of the) famous theorem by Riesz and Thorin on the interpolation of linear operators.

Sophie Morier-Genoud (Université Pierre et Marie Curie, Paris 6): Arithmetic and combinatorics of Conway–Coxeter frieze patterns

Abstract. Continued fractions give an alternative way, comparing to the decimal system, to represent a real number as a sequence of integers. There are various approaches to this classical subject, and applications to many different areas of mathematics can be found in the literature. In this course we will develop a simple combinatorial approach based on such basic notions of combinatorics as triangulations of polygons. One of the main ingredients is the Farey graph of rational numbers. We will explore the connections with the beautiful and not very well-known notion of Conway–Coxeter frieze patterns.

Enrico Scalas (University of Sussex): Monte Carlo simulations of anomalous random walks

Abstract. The random walk can be seen as a process in which a particle moves randomly in space. We shall consider random walks in a one-dimensional space. So the particle will be able to move just to the left or to the right of its current position on the real line. We shall assume that the particle stays in its current position for a random waiting time and then instantaneously jumps either to the left or to the right with the jump size being a real random variable.

Monte Carlo simulations are computer simulations using pseudo-random number generators. Using a language called R, we shall write a program generating "anomalous" random walks where both the waiting times and the jumps have a distribution

with power-law tails making extreme values more likely than with exponential and Gaussian distributions. Using this program, we shall explore some interesting conjectures on the behaviour of the random walk under rescaling of waiting times and jumps. It turns out that these conjectures can be rigorously proven.

Sarah Whitehouse (University of Sheffield): Combinatorics of Young tableaux and symmetric groups

Abstract. The course will start with basic definitions of partitions, Young diagrams and Young tableaux. The hook and determinant formulas for the number of standard Young tableaux of a given shape will be presented, probably with sketch proofs. I will explain the relationship between tableaux and permutations and sketch the relevance to representations of the symmetric group. Throughout there will be a strong emphasis on examples and calculations, rather than abstract representation theory. Many examples will be worked out in detail for small n, with exercises leading the students through further examples.

Colloquia

Christina Cobbold (University of Glasgow): Mathematical modelling tools for vector borne diseases

Abstract. Many well known diseases such as malaria, dengue, Zika, and West Nile Virus are transmitted to humans via an insect vector and are collectively referred to as vector borne diseases. All these diseases pose major public health concerns. The insect vectors can carry the disease and pass the infection to new human or animal hosts via an insect bite. Climate change has been proposed as a likely driver of past and future geographical expansion of these diseases, however the complex ecology of insect populations makes understanding the likely impacts of climate change on vector borne diseases challenging. Tools that can predict when insect vector numbers will be high can allow us to plan management strategies to mitigate against future disease outbreaks. In this talk I will show how differential equations can be used to describe how insect abundance and disease risk change in time. I will show how we can use these mathematical models to make predictions about the threat of West Nile Virus in the UK.

Anna Felikson (Durham University): Quiver mutations and triangulated surfaces

Abstract. Quiver mutations arise within the recent theory of Cluster Algebras that was introduced by Fomin and Zelevinsky in 2002, and since then turned out to be connected to numerous different fields in mathematics and mathematical physics. In this talk we will introduce quiver mutations and will show some connections to triangulated surfaces. The work is joint with Pavel Tumarkin and Michael Shapiro. Quiver mutations arise within the recent theory of Cluster Algebras that was introduced by Fomin and Zelevinsky in 2002, and since then turned out to be connected to numerous different fields in mathematics and mathematical physics. In this talk we will introduce

quiver mutations and will show some connections to triangulated surfaces. The work is joint with Pavel Tumarkin and Michael Shapiro.

Michael Grinfeld (University of Strathclyde): Coagulation-fragmentation modelling

Abstract. Coagulation fragmentation phenomena arise in a wide variety of contexts, from aerosol dynamics to fish schools to planetoid formation. They are frequently described by infinite systems of ordinary differential equations and questions of existence and behaviour of solutions are quite challenging, requiring tools from many areas of mathematics. I will survey this area of research and will discuss results that seem to me counterintuitive and surprising.

Ian Leary (University of Southampton): Countability, groups, subgroups and finiteness properties

Abstract. What might it mean to give a finite description of an infinite group? Does this constraint on a group tell us anything about its subgroups? I shall discuss three possible answers to this pair of questions, invoking theorems from 1949, 1961 and 2018.

Felipe Rincon (Queen Mary University of London): A glimpse of tropical geometry

Abstract. Tropical geometry is geometry over the tropical numbers, where multiplication is replaced by addition and addition is replaced by minimum. One can "tropicalise" algebro-geometric spaces in this way and get polyhedral spaces as a result, which can be combinatorially studied. In this talk I will give a gentle introduction to this topic, and present a couple of applications in different fields of mathematics.

Melanie Rupflin (University of Oxford): Asymptotic behaviour of gradient flows

Abstract. Many interesting geometric objects are characterised as minimisers or critical points of natural geometric quantities such as the length of a curve, the area of a surface or the energy of a map.

If we want to deform a given geometric object in a way that is designed to change the initial object towards such an optimal state, it is hence natural to move in the direction of the negative gradient of the corresponding functional.

In this talk we will discuss some aspects of a well known geometric flow, the so called harmonic map flow, which is defined as the gradient flow of the Dirichlet energy $E(u) = \frac{1}{2}\int_M |\nabla u|^2$. We will in particular focus on the possible behaviour of solutions as time tends to infinity and the question of whether one can expect such a gradient flow to converge to a minimiser of the underlying energy.

Caroline Series (University of Warwick): Markov numbers and the free group on two generators

Abstract. Markov numbers are the integer solutions of the equation $x^2 + y^2 + z^2 = 3xyz$. Markov proved that the solutions can be arranged around the vertices of a trivalent tree, which leads to a tantalising unsolved problem called the uniqueness conjecture. Another well known classical theorem says that the possible sets of generators of the free group on two generators can be arranged in a similar way. It turns out that these two results are closely related. After explaining how all this works, we will go on to some modern work involving complex number solutions of the Markov equation where there are some very nice results but also many unanswered questions.

Arkadiusz Wiśniowski (University of Manchester): Measuring migration by using traditional and new forms of data

Abstract. Having up-to-date information about the nature and extent of migration within the EU is important for policy making, such as labour market policy or social services. However, timely and reliable statistics on the number of EU citizens residing in or moving across other Member states are difficult to obtain. Official statistics on EU movers are developed by national offices of statistics and published by Eurostat, but they come with a considerable time lag of about two years. With the rise of the Internet, new data sources offer opportunities to complement traditional sources for EU mobility statistics. In particular, the availability of high quantities of data derived from social media has opened new opportunities. Therefore, we propose a statistical model that integrates data on migrant stocks within the EU from traditional sources such as census, population registers and Labour Force Survey, with new forms of data derived from Facebook. Then, we investigate the potential of the model to facilitate "now-casting," that is, providing nearly real-time estimates that can serve as early warnings about changes in EU mobility. The model provides measures of uncertainty for the estimates of migrant stocks.

	Monday 12th	Tuesday 13th	Wednesday 14th	Thursday 15th	Friday 16th
09:45 - 10:00	**Opening of the School**				
10:00 - 11:00	Sarah Whitehouse *Lecture 1*	Sarah Whitehouse *Lecture 3*	Sarah Whitehouse *Lecture 5*	Sophie Morier-Genoud *Lecture 3*	Ivan Cheltsov *Lecture 5*
11:00 - 11:30	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee
11:30 -12:30	Sophie Morier-Genoud *Lecture 1*	Sophie Morier-Genoud *Lecture 2*	Ivan Cheltsov *Lecture 1*	Sophie Morier-Genoud *Lecture 4*	Sophie Morier-Genoud *Lecture 5*
12:30 - 14:30	Lunch & Free Time	Lunch & Free Time	Lunch & Free Time	Lunch & Free Time	Lunch & Free Time
14:30 - 15:30	Sarah Whitehouse *Lecture 2*	Sarah Whitehouse *Lecture 4*	Ivan Cheltsov *Lecture 2*	Ivan Cheltsov *Lecture 3*	Anna Felikson *Colloquium*
15:30 - 16:00	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee	
16:00 - 17:00	Caroline Series *Colloquium*	Felipe Rincon *Colloquium*	Christina Cobbold *Colloquium*	Ivan Cheltsov *Lecture 4*	
17:00 - 17:30					

	Monday 19th	Tuesday 20th	Wednesday 21st	Thursday 22nd	Friday 23rd
10:00 - 11:00	Enrico Scalas *Lecture 1*	Enrico Scalas Lecture 3	Enrico Scalas *Lecture 5*	Tobias Kuna *Lecture 2*	Tobias Kuna *Lecture 5*
11:00 - 11:30	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee
11:30 -12:30	Carlo Marinelli *Lecture 1*	Carlo Marinelli *Lecture 2*	Carlo Marinelli *Lecture 3*	Carlo Marinelli *Lecture 4*	Carlo Marinelli *Lecture 5*
12:30 - 14:30	Lunch & Free Time	Lunch & Free Time	Lunch & Free Time	Lunch & Free Time	Lunch & Free Time
14:30 - 15:30	Enrico Scalas *Lecture 2*	Enrico Scalas *Lecture 4*	Tobias Kuna *Lecture 1*	Tobias Kuna *Lecture 3*	Melanie Rupflin *Colloquium*
15:30 - 16:00	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee	Virtual Tea / Coffee	
16:00 - 17:00	Ian Leary *Colloquium*	Michael Grinfeld *Colloquium*	Arkadiusz Wiśniowski *Colloquium*	Tobias Kuna *Lecture 4*	
17:00 - 17:30					