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COLLOQUIUM

Caroline Series (Warwick)

Simple curves on hyperbolic surfaces

Abstract

Hyperbolic geometry is geometry in which the sum of angles in a triangle can be anything less than 180 degrees. Just as you can make a rectangle by gluing together opposite sides of a Euclidean rectangle, so you can make more complicated surfaces by gluing sides of hyperbolic polygons. This allows one to use methods of hyperbolic geometry to study closed loops on surfaces. The basic building blocks are closed loops with no self intersections, called simple curves.

In this talk we will discuss some of the features of simple curves on surfaces. Using the geometry on the surface, we can restrict to simple curves which are also geodesics. Such geodesics have been the object of study for many years. On a Euclidean torus, there is a simple geodesic through every point on the surface. By contrast, on hyperbolic surfaces they are very sparse – just how sparse we shall see in the lecture. We will go on to some counting questions, starting with some rather simple estimates which were turned into beautiful theorems by Maryam Mirzakhani – the first female Fields Medallist who sadly died in 2017 aged only 40 – and which are still the subject of ongoing research.