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Hopf algebras, Frobenius functors and the Structure Theorem of Hopf Modules

Abstract

We report on some recent advances concerning how Frobenius functors naturally intervene in the study of Frobenius Hopf algebras.

Frobenius algebras originally appeared in representation theory at the beginning of the XX century, but the interest in these structures has been recently renewed due their connection with 2-dimensional Topological Quantum Field Theories and monoidal categories.

Hopf algebras, for their part, are the backbone of the algebraic approach to many questions in geometry, topology, representation theory, mathematical physics, and they are nowadays recognized as the algebraic counterpart of groups, even in situations where “groups” do not strictly make sense (such as non-commutative geometry).

In a recent pair of preprints, we reveal the existence of a deep connection between Frobenius functors on the one hand (a categorical extension of the Frobenius algebra notion) and Hopf algebras and their categories of Hopf modules on the other. Namely, we will see how being Frobenius for the free (two-sided) Hopf module functor $- \otimes B$ (the main ingredient of the celebrated Structure Theorem of Hopf modules) is related to being a Hopf algebra for the bialgebra B and how this can be connected with the theory of Hopf and Frobenius monads.