Symmetry of constrained minimizers of a Cahn-Hilliard energy on the torus

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Abstract. We are interested in symmetry of constrained minimizers of a Cahn-Hilliard energy on the torus. Steiner symmetrization is a natural tool in such a setting, and it is easy to use Steiner symmetrization to show that there exist minimizers with the symmetries of the torus. In this paper, we show that in fact any constrained minimizer is (up to a shift) equal to its Steiner symmetrization. To do so, we formulate general sufficient conditions for a function on the torus to be equal to its Steiner symmetrization. Applying the result to our Cahn-Hilliard model, we obtain in particular that the superlevel sets of minimizers are simply connected. In two dimensions, we use this together with the Bonnesen inequality to derive a new bound on the sphericity of minimizers, which rules out phenomena such as "tentacles." An even simpler rearrangement is the two-point rearrangement or polarization of a function. In general two-point rearrangements give weaker results than symmetrization. For the Cahn-Hilliard problem, however, we will obtain from two-point rearrangements that a minimizer is equal to its reflection with respect to some hyperplane and from here deduce strict monotonicity properties.

This is joint work with Maria G. Westdickenberg (RWTH Aachen University) and Michael Gelantalis, (University of Tennessee).