Allen-Cahn equation with a non-decreasing constraint

Goro Akagi,
Mathematical Institute, Tohoku University
goro.akagi@tohoku.ac.jp

Abstract

This talk is concerned with a constraint Allen-Cahn equation whose solution is constrained to be non-decreasing. The study of such a strongly irreversible evolution equation is motivated from Damage Mechanics, where phase parameters represent the degree of damage of specimen and is therefore supposed to be monotone. The first half of this talk is devoted to the Cauchy-Dirichlet problem on bounded domains. More precisely, we shall overview some recent results on well-posedness, qualitative properties and asymptotic behavior of solutions. In particular, the non-decreasing constraint prevents energy-dissipation of the dynamics and smoothing effect of solutions, which are essential features of the classical Allen-Cahn equation. Indeed, one can prove non-existence of global attractor in any Lebesgue and Sobolev spaces. On the other hand, the equation still involves a gradient structure and it hence exhibits a *partial* energy-dissipation and smoothing effect. In this part, we shall see how to configure a function space setting and how to extract such a partial energy-dissipation and a smoothing effect. This part is based on a joint work with M. Efendiev (München). In the second half of this talk, we shall deal with traveling wave solutions in the one-dimensional case. Indeed, the constraint Allen-Cahn equation also exhibits traveling wave dynamics just like the classical Allen-Cahn equation does. On the other hand, it presents very difference appearances due to the presence of the non-decreasing constraint. For instance, traveling wave solutions may have non-zero velocity even though the double-well potential is balanced, and moreover, profile function and velocity are not uniquely determined. In this part, we shall first discuss existence of such traveling wave solutions. Next, we shall show that such a traveling wave dynamics is generated by some novel free boundary problem, and then, a motion equation of the free boundary will be also formulated. This part is based on a joint work with C. Kuehn (München).