The heat equation with critical growth in the gradient respect to the Hardy potential

Ireneo Peral Alonso

Departamento de Matemáticas, UAM, Campus de Cantoblanco 28049 Madrid, España
[ireneo.peral@uam.es]

2000 Mathematics Subject Classification. 35K55, 35K65, 35B05, 35B40, 46E30

We deal with the following parabolic problem

$$u_t - \Delta u = |\nabla u|^p + \lambda \frac{u}{|x|^2} + f, \quad u > 0 \text{ in } \Omega \times (0, T),$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 3$, is a bounded regular domain such that $0 \in \Omega$, $p > 1$, $\lambda \geq 0$ and $f \geq 0$ are in a suitable class of functions.

There are deep differences with respect to the behavior of the heat equation $(\lambda = 0)$. The main features in the talk are the following.

- If $\lambda > 0$, there exists a critical exponent $p_+(l)$ such that for $p \geq p_+(\lambda)$, there is no solution for any nontrivial initial datum.
- $p_+(\lambda)$ is optimal in the sense that, if $p < p_+(\lambda)$ there exists solution for suitable data.
- If $\Omega \equiv \mathbb{R}^N$, i.e., we consider the Cauchy problem, then there exists a Fujita type exponent $F(\lambda) < p_+(\lambda)$ in the sense that for $1 < p < F(\lambda)$, then independently of the initial datum, any solution blows-up in a finite time. Notice that this is a deep difference with the case of the heat equation $(\lambda = 0)$ with a gradient term, for which given any $p > 1$, there exist small initial data such that there exists a global solution.

This is a joint work with

- Boumediene Abdellaoui, *Département de Mathématiques, Université Aboubekr Belkaïd, Tlemcen, Tlemcen 13000, Algeria.*
- Ana Primo, *Departamento de Matemáticas, U. Autonoma de Madrid, 28049 Madrid, Spain.*