Boundary singularities for solutions of non-monotone nonlinear elliptic equations

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Abstract

Let $\Omega \subset \mathbf{R^N}$ be a smooth domain, $x_0 \in \partial\Omega$ and $q \geq (N+1)/(N-1)$. We study the behavior near x_0 of any positive solution of (E) $-\Delta u = u^q$ in Ω which coincides with some $\zeta \in C^2(\partial\Omega)$ on $\partial\Omega \setminus \{x_0\}$. We prove that, if $(N+1)/(N-1) < q < (N+2)/(N-2)$, u satisfies $u(x) \leq C|x-x_0|^{-2/(q-1)}$ and we give the limit of $|x-x_0|^{2/(q-1)}u(x)$ as $x \to x_0$. In the case where $q = (N+1)/(N-1)$, u satisfies $u(x) \leq C|x-x_0|^{1-N} (\ln(1/|x|))^{(1-N)/2}$ and a corresponding precise asymptotic is obtained. We also study some existence and uniqueness questions for related equations on spherical domains.