

Parabolic problems with dynamic boundary conditions

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Abstract

An existence theory for local solutions of a parabolic problem under a dynamical boundary condition $\sigma u_t + u_n = 0$ is developed and a spectral representation formula is derived. It relies on the spectral theory of an associated elliptic problem with the eigenvalue parameter both in the equation and the boundary condition. The well-posedness of the parabolic problem holds in some natural space only if the number of negative eigenvalues is finite. This depends on the parameter σ in the boundary condition. If $\sigma \geq 0$ the parabolic problem is always well-posed. For $\sigma < 0$ it is well-posed only if the space dimension is 1 and ill-posed in space dimension ≥ 2 . By means of the theory of compact operators the spectrum is analyzed and some qualitative properties of the eigenfunctions are derived. An interesting phenomenon is the “parameter-resonance”, where for a specific parameter-value σ_0 two eigenvalues of the elliptic problem cross. Depending on the time some qualitative properties will be discussed.