Singular solutions of the nonlinear heat equation

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We consider the Cauchy problem

$$\begin{aligned} &u_t = \Delta u + u^p, \qquad x \in \mathbb{R}^n, t > 0, \\ &u(x,0) = u_0(x), \end{aligned}$$

where p is supercritical in the sense that $n \geqslant 11$ and $p \geqslant p_{JL} = \frac{n-2\sqrt{n-1}}{n-4-2\sqrt{n-1}}$. The goal of this talk is to discuss the asymptotic stability of a singular steady state

$$v_\infty(x) = \left(\frac{2}{p-1}\left(n-2-\frac{2}{p-1}\right)\right)^{\frac{1}{p-1}} |x|^{-\frac{2}{p-1}}$$

in the weighted L^r- norm for $1 \leqslant r \leqslant \infty$. In our reasoning we use estimates of the fundamental solution of the equation

$$u_t = \Delta u + \frac{\lambda}{|x|^2} u$$

obtained recently by Milman and Semenov.