REACTION-DIFFUSION EQUATIONS ON MANIFOLDS

GARETH CLEWS
BRUNEL UNIVERSITY

In 1952 Turing proposed that homogeneous steady states could be destabilised by adding diffusion to a system and that this effect could explain the emergence of patterns in the system. One example of such Turing patterns are localised peaks in the concentration of a chemical substance known as a morphogen. These models could explain pattern formation in many biological contexts, for example they could give a caricature for the development of a complex organism from a single cell which could be explained by a series of bifurcations of patterns with increasing complexity, a process which Turing termed morphogenesis.

Gierer-Meinhardt in 1972 put forward their reaction-diffusion system of activator-inhibitor type. We study this model analytically in the case of one space dimension. The literature to date considers this on an interval with Neumann boundary values. We present some interesting new behaviour for these equations on the unit circle.

We provide a stability analysis for both symmetric and non-symmetric spike solutions where there are one or two possible heights of the spike solutions, respectively. In the first case this is a global analysis and in the latter it is local to the bifurcation point of the large ($O(1)$) eigenvalues.