Positive semigroups generated by degenerate second-order differential operators

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This talk in concerned with initial boundary problems associated with a wide class of degenerate second-order differential operators in the framework of weighted continuous function spaces on a real interval J.
Our approach is essentially based on semigroup theory.
We will show that degenerate second order differential operators of the form

$$Lu := \alpha u'' + \beta u' + \gamma u$$

defined on maximal type domain and respectively on Wentzell type domain, under suitable conditions on the coefficients α, β and γ, generate positive strongly continuous semigroup on the space

$$E^w(J) := \{f \in C(J) : wf \in E(J)\}$$

where w is a weight function on J, i.e. $w \in C_b(J)$ and $w(x) > 0$ for every $x \in J$, and

$$E(J) := \Big\{ f \in C(J) : \text{there exists } \lim_{x \to r_i^{\pm}} f(x) \in \mathbb{R} \ \text{ for every } i = 1, 2 \\ \text{such that } r_i \notin J \ \Big\}$$

where $r_1 := \inf J \in \mathbb{R} \cup \{-\infty\}$ and $r_2 := \sup J \in \mathbb{R} \cup \{+\infty\}$.
Moreover we investigate under which conditions the generated semigroup is the transition semigroup associated with a suitable Markov process.
Our results apply, in particular, to the differential operator associated with the Black-Scholes equation; as a consequence we state some new results about existence, uniqueness and continuous dependence on the initial data of the solutions of the evolution problem related to the Black-Scholes equation.