Lipschitz solutions for a 1-dimensional fast diffusion equation

Nicole Schadewaldt, University of Cologne (Köln), Germany

ABSTRACT: I consider the following 1-dimensional diffusion equation

$$u_t = \frac{u_{xx}}{u_x} = [\mathrm{sign}(u_x)\log(|u_x|)]_x\,, \tag{1}$$

which is singular for $u_x = 0$. A. Rodriguez and J.L. Vazquez have studied equations of this type in [1]. They show that for non-monotone initial data u_0, when the singularity cannot be avoided, no C^1-solution of (1) exists.

However, with a method developed by K. Zhang for the 1-dimensional Perona-Malik equation in [2], I am able to prove the existence of infinitely many $W^{1,\infty}$-solutions for the Dirichlet-problem of equation (1) and initial data $u_0 \in C^{2+\alpha}$.

Such solutions are constructed by adding piecewise affine functions to an approximate solution and thereby creating a converging sequence. The reformulation of the problem as a differential inclusion is the key to the convergence result.

References

[1] A. Rodriguez, R.L. Vazquez *Obstructions to existence in fast-diffusion equations*, Journal of Differential Equations 184(2):348-385, 2002.

[2] K. Zhang *Existence of infinitely many solutions for the one-dimensional perona-malik model*, Calculus of Variations and Partial Differential Equations, 26(2):171-199, 2006.