

Local smoothing effects, positivity, and Harnack inequalities for the fast p -Laplacian equation

Abstract

We study qualitative (regularity) properties of local weak solutions of the fast p -Laplacian equation, $u_t = \Delta_p u$, with $1 < p < 2$. We introduce first a new and interesting general local inequality for gradients of local weak solutions, then, using it, we establish local upper bounds of Herrero-Pierre type for local norms of the solutions and of their gradients. These bounds imply a new local $L^r - L^\infty$ smoothing effect. In the second part, we establish local lower bounds for the solutions, in form of positivity estimates in small balls and annuli. We combine these local lower and upper bounds in different forms of intrinsic Harnack inequalities, which are completely new in the subcritical range of the fast p -Laplacian equation, that is when $1 < p < p_c = 2n/(n+1)$.

Joint work with Matteo Bonforte and Juan Luis Vázquez.