HARNACK INEQUALITY FOR GENERALIZED MEHLER SEMIGROUP

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Let $\mathcal{H}$ be a separable Hilbert space and Z_t a $\mathcal{H}$-valued Lévy process on a probability space with measure $\mathbb{P}$. Suppose that Z_t has a Gaussian part with covariance R. Consider the following generalized Langevin equation

$$dX_t = AX_t dt + dZ_t, \quad X_0 = x \in \mathcal{H}$$

where $\{S_t\}_{t\geq 0}$ is a strongly continuous contraction semigroup on $\mathcal{H}$ generated by A. The associated transition semigroup of X_t is given by

$$P_t f(x) = \int_{\mathcal{H}} f(S_t x + y)\mu_t(dy),$$

here μ_t is the law of $\int_0^t S_{t-s} dZ_s$.

P_t is a generalized Mehler semigroup. Under some slight conditions, any generalized Mehler semigroup is a transition semigroup of the above form (see [2, 3]).

By the method in [1], i.e., a combination of the coupling argument and Girsanov transformation, we can prove the following Harnack inequality.

Assume for $T > 0$, $u \in [0, T]$, and every $x, y \in \mathcal{H}$, $S_u(x - y) \subset R^{1/2}(\mathcal{H})$. Then for any $\alpha > 0, \beta > 0$ satisfying $\alpha^{-1} + \beta^{-1} = 1$, $f \in \mathcal{C}_b^+(\mathcal{H})$, we have

$$\frac{[P_T f(x)]^\alpha}{P_T f^\alpha(y)} \leq \exp\left\{ \frac{\beta}{2} \frac{\int_0^T |R^{-1/2} S_u(x-y)\xi_u|^2 du}{\left(\int_0^T \xi_u du\right)^2} \right\}.$$

Here ξ is a positive continuous function on $[0, +\infty)$.

This is a generalization of a main result in [4] in which only the second order Harnack inequality was able to be established.

References

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[4] Michael Röckner and Feng-Yu Wang (communicated by Leonard Gross). Harnack and functional inequalities for generalized Mehler semigroups. *J. Funct. Anal.*, 203:237–261, 2003.