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On a more general nonlinear eigenvalue problem.

Abstract. Let $N \in \mathbb{N}$, $p \in (1, +\infty)$, $q \in (1, p^*)$, where $p^* = Np/(N-p)$ if $p < N$ and $p^* = +\infty$ if $p \geq N$. We consider the quasilinear PDE

$$-\mathrm{div}(|\nabla \mathrm{u}|^{\mathrm{p}-2}\nabla \mathrm{u}) = \lambda \|\mathrm{u}\|^{\mathrm{p}-\mathrm{q}}_{\mathrm{L}^{\mathrm{q}}(\Omega)}|\mathrm{u}|^{\mathrm{q}-2}\mathrm{u}, \qquad \text{(P)}$$

in a domain of finite Lebesgue measure in $\mathbb{R}^N$, with the Dirichlet conditions $u = 0$ on the boundary $\partial\Omega$. We will point out that several results known for the case $q = p$, when (P) reduces to the typical eigenvalue problem for the p-laplacian, extend to the more general case $q \in (1, p^*)$. It turns out that (P) can be regarded as well as an eigenvalue problem. In particular, we will focus on surveying some uniqueness results concerning the first eigenvalue

$$\lambda_1(p, q) = \min_{0 \neq u \in W^{1,p}_0(\Omega)} \frac{\int_\Omega |\nabla u|^p dx}{(\int_\Omega |u|^q dx)^{p/q}}.$$

In the "subtypical" cases $q \leq p$ the first eigenvalue is simple for all domain Ω of finite Lebesgue measure, whereas, if $q > p$, it this false in general. Nevertheless, if Ω is a ball in $\mathbb{R}^N$, then $\lambda_1(p, q)$ is simple for all $q \in (1, p^*)$.