CRITICALITY THEORY FOR SCHRÖDINGER OPERATORS

EXERCISES WEEK 3 & 4

If you need this exercises to be assessed submit to `v.moroz@swansea.ac.uk` by 1pm on 24 February

Exercise 1. Let H be a Hilbert space and $\|\cdot\|_H = \sqrt{\langle\cdot,\cdot\rangle_H}$. Show that $\|u\|_H^2$ is a strictly convex function on H.

Exercise 2. Let $N \geq 2$ and $\sigma < 2$. Use scaling argument to show that the inequality

$$\int_{B_R} |\nabla\varphi|^2 \geq c \int_{B_R} \frac{\varphi^2}{|x|^\sigma} \qquad \forall \varphi \in C_c^\infty(\mathbb{R}^N)$$

fails for any $c > 0$.

Exercise 3. Show that $u = 1 - |x|^{-1} \in D_0^1(\mathbb{R}^2 \setminus \bar{B}_1)$.

Exercise 4. Let $N \geq 3$ and $c < C_H = \frac{(N-2)^2}{4}$. Find minimal positive solution u_1 and Agmon's positive solution u_0 to the equation

$$-\Delta u - \frac{c}{|x|^2} u = 0 \quad \text{in } \mathbb{R}^N \setminus \bar{B}_1.$$