CRITICALITY THEORY FOR SCHRÖDINGER OPERATORS

EXERCISES WEEK 5 & 6

If you need this exercises to be assessed submit to `v.moroz@swansea.ac.uk` by 1pm on 10 March

Exercise 1. Let $N \geq 3$, $\varepsilon > 0$ and $c \in \mathbb{R}$. Show that if $u > 0$ is a super–solution to

$$-\Delta u - \frac{c}{1 + |x|^{2+\varepsilon}} u = 0 \quad \text{in } \mathbb{R}^N \setminus \bar{B}_1$$

then

$$\liminf_{|x| \to \infty} \frac{u(x)}{|x|^{2-N}} > 0 \qquad \text{and} \qquad \liminf_{|x| \to \infty} u(x) < +\infty.$$

Exercise 2. Let $N \geq 3$, $p > 1$ and $s < 2$. Show that

$$-\Delta u = |x|^{-s} u^p \quad \text{in } \mathbb{R}^N \setminus \bar{B}_1$$

has no positive super–solutions if $p \leq \frac{N-s}{N-2}$.