

CRITICALITY THEORY FOR SCHRÖDINGER OPERATORS

EXERCISES WEEK 7 & 8

If you need this exercises to be assessed submit to v.moroz@swansea.ac.uk by 1pm on 24 March

Exercise 1. Show that for every $\mu < 1/4$, the function $u = \delta_\Omega^{1/2}$ is a local super-solution to

$$-\Delta u - \frac{\mu}{\delta_\Omega^2} u = 0 \quad \text{in } \Omega_{\bar{\rho}},$$

for a small $\bar{\rho} > 0$. Conclude from this that

$$\int_\Omega |\nabla \varphi|^2 \geq \frac{1}{4} \int_\Omega \frac{\varphi^2}{\delta_\Omega^2} \quad \forall \varphi \in C_c^\infty(\Omega_\rho).$$

Exercise 2. Let $p > 1$ and $\mu < 0$. Use the Keller-Osserman bound and Phragmen-Lindelof principle to show that

$$-\Delta u - \frac{\mu}{\delta_\Omega^2} u + u^p = 0 \quad \text{in } \Omega$$

has no *large* positive sub-solutions if $p < 1 - \frac{2}{\beta_-}$, here $\beta_- < 0$ is the smallest root of $-\beta(\beta - 1) = \mu$.